# CAPITAL AND OIL IN THE GLOBAL ECONOMY: OPTIMAL INVESTMENT AND FINANCIAL OPENNESS

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## ***ABSTRACT***

The global economy is energy-dependent because energy consumption is comprehensive, capital and energy are to a large extent complementary production factors, and international trade in the energy-carriers is highly specialized. A two-country model of this paper reflects these features and emphasizes a dynamic interdependence between capital accumulation and oil stock extension by oil-consuming and oil-producing economies. Financial openness under conditions of energy-dependence is necessary for optimal investment in the production factors separated by the national border. The oil price plays in this model a key role underlying the equalization of returns and the determination of optimal factor structure in the global economy. The model extension to the case of uncertainty in oil demand demonstrates that cross-country investments eliminate negatively correlated country-specific risks and allows households to build asset portfolios composed of non-diversifiable global risks which are the same for both economies.

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## Introduction

The structure of the modern global economy is characterized by a high degree of dependence of the advanced countries on the energy supplied by the less developed ones. On the one hand, energy and oil, as the main energy source, still remain the essential inputs for most industries. The essence of this technological dependence lies in a high degree of complementarity between physical capital and energy. Smith (2009) points at a very low oil demand elasticity equal to – 0.05 in the short run indicating a narrow domain for substitution of energy inputs. On the other hand, the global oil and natural gas reserves are distributed very unevenly between developed and less developed economies, and actually concentrate in the few oil-rich countries. For instance, OPEC and three former Soviet republics – Azerbaijan, Kazakhstan and Russia – produce together only 5.3 per cent of the world GDP but are endowed with 77.3 per cent of the total world oil reserves (according to International Energy Agency information).

A high degree of specialization in the global energy production and consumption results in the absence of trade barriers against imports of hydrocarbons. Essential policy barriers are, nevertheless, imposed against cross-border investment flows associated directly or indirectly with the activity of the world energy sector. A tendency of financial protectionism manifested strongly before the current global crisis, at the period of soaring world oil price. Some politicians in the developed countries advanced defensive measures against the sovereign oil funds engaged in strategic investment abroad that, supposedly, threatened national security. This tendency became even more prominent as the energy-dependence had got the status of a national security problem in the developed countries. Another side of the very same tendency is the strategy of resource nationalism exercised in some oil-exporting countries to maintain control over national resources. Financial protectionism in both cases is motivated merely by political arguments like claims to prevent the threats of losing strategic controls.

From a pure economic viewpoint, financial protectionism should be regarded as a doubtful policy. The goal of this paper is to articulate some theoretical arguments supporting this thesis and emphasizing the issue of global oil-dependence. Our basic premise is that both production capital and oil reserves can be extended through investment. It is important that the extension of oil reserves in the oil-dependent world economy should be closely linked to the accumulation of productive capital. This link is missing in the absence of international financial markets resulting in the impediments to optimal intertemporal choice and international risk sharing and leading to misallocation of global investments. Free trade in goods under perfect markets should bring about optimal resource allocation, given the stocks of factors. But the relative size of these stocks – the ratio of global capital to oil reserves – may be suboptimal if households in oil-producing/consuming countries are constrained to invest only in production factors concentrated within national borders. Financial openness should be regarded, the real-life global tendencies notwithstanding, as an important condition for optimal capital growth.

To formalize this kind of reasoning we suggest a two-country neoclassical model of trade and factor accumulation with oil as a production input complementary to physical capital. One of the countries is endowed with a capital stock and a final good manufacturing technology, while another one – with an oil stock and an oil extraction technology. The economies are completely specialized in production and trade in which the final good is exchanged for oil (J7 and OPEC is a good example of such exchange). Following the vision of Adelman (1990)[1], we ignore the ultimate depletion of the world oil reserves which is indefinitely distant. The oil stock is assumed to be an inexhaustible factor playing in our model a dual role. It is a reservoir of oil and, at the same time, a production factor in oil extraction similarly to capital in the final good manufacturing.[2] This property implies that the equilibrium oil price is the sum of the marginal costs of oil stock maintaining and oil extraction. The former is constant, while the latter is increasing with *the global factor structure* – the world ratio of capital to oil stock – which is predetermined by investment decisions made previously by households in both countries.

The oil price is a key element in the model mechanism of global dynamic. Both the capital and the oil stock can be extended through investment that predetermine the next period oil price which, in turn, defines the expected returns on investment in both production factors. Optimal investment policy requires cross-country equalization of the marginal rates of substitution for households and, consequently – of the marginal returns on investment in capital and oil stocks. Individual household investment decisions are shown to be optimal provided that the factor returns are equalized. This is ensured in our model by an implicit

[1] Adelman's (1990) critique of "the Hotelling theory" is based on the assumption of fixed stock of mineral resource: "There is no such thing. The total mineral in the earth is irrelevant non-binding constraint. If expected finding-development costs exceed the expected net revenues, investment dries up, and the industry disappears. Whatever is left in the ground is unknown, probably unknowable, but surely unimportant; a geological fact of no economic interest" (op. cit., p. 1).

[2] This kind of oil extraction technology has been used, for instance, by Devarajan and Fisher (1982) in a two-period model of natural resource extraction and reserve exploration under uncertainty.

forward optimal oil price which defines the optimal global factor structure in the next period. Under financial autarky the returns on capital and oil stock, generally, differ for a transitory equilibrium path. As it turns out for the deterministic case, opening of the international bond market is sufficient for equalization of returns and optimal consumption smoothing and investment.

The model is extended to the case of uncertainty in the oil-to-capital input coefficient causing fluctuations of oil demand. Under financial autarky the returns on capital and oil stock are negatively correlated, as well as the countries' incomes, indicating non-optimality of resource allocation in the world economy. A necessary condition for optimal risk sharing is the cross-country equalization of marginal rates of substitution across countries implying equalization of expected risk-adjusted returns on the production factors. The international risk sharing results in optimal oil stock investment which is determined via the implicit forward oil price. In the case of state-contingent securities this price eliminates the negatively correlated risks in the factor returns through mutual insurance between the economies.

This inference should be contrasted with an earlier finding by Cole and Obstfeld (1991) that asset trade is redundant under free trade in goods. Using a Lucas-type (1982) general equilibrium two-country framework with investment in production factors and output shocks they showed for some important "knife-edge" cases that financial autarky results in the same resource allocation as an optimal plan or trading equilibrium with complete markets. Output shocks in the Cole-Obstfeld model cause fluctuations in the terms of trade that automatically pool risks, since a country's terms of trade are negatively correlated with growth in its export sector. For certain parameter choices the terms-of-trade responses alone provide perfect insurance against output shocks through complete risk sharing between countries.[3] On the contrary, the terms of trade in our model aggravate the effects of oil demand shocks on the world income distribution. The reason is that, under a given oil stock, the marginal oil extraction cost and, hence, the oil price is increasing under a positive oil demand shock. The positive terms-of-trade effect persists under financial autarky due to complementarity between energy and capital, and this is the main reason why financial

[3] In the recent paper Heathcote and Perri (2009) demonstrated the similar result based on the Backus, Kehoe and Kydland (1995) framework. They showed that changes in international relative prices provide some insurance against country-specific shocks, and due to this portfolio choice does not play the key role in risk-sharing. Home bias in the Heathcote-Perri model arises because relative returns to domestic stocks move inversely with relative labor income in response to productivity shocks and deliver thereby hedging against these shocks. This result is contrasted to the earlier Baxter and Jermann (1997) finding that the international diversification puzzle is deepened due to the implications of non-traded human capital and non-diversifiable labor income.

openness should be viewed as contributing to the growth of the energy-dependent global economy.

We show that international asset trade can be sufficient for a cross-country exchange of risks that mitigates or even eliminates the shock-aggravating terms-of-trade effects even without market completeness. Opening of borders for direct and indirect foreign investment results in the equalization of risk-adjusted expected returns on production factors. Though the interstate marginal rates of substitution still vary between the countries, the global factor structure turns out to be near optimal for a small oil extraction rate (the ratio of oil extraction to oil stock) that indicates a low degree of oil dependence. This is a consequence of asset trade occurring through indirect cross-country investment in factors and bringing about a positive correlation of risk premium in the factor returns. The structure of a risky *benchmark asset portfolio* corresponds to the global factor structure and is the same for both economies. Households are rewarded by the benchmark portfolio if the oil price falls, but this reward is larger on average for a higher oil price. As a result, the asset trade provides incentives for households in both countries to make risky investment under an upsurge of oil demand.

The next section presents the basic equilibrium model of the global economy and examines financial autarky equilibrium under certainty. The deterministic model captures the essence of the problem and is presented for the sake of expositional convenience. Section 2 deals with a global planner problem and conditions of optimal investment. Section 3 extends the basic model to uncertain oil demand and concerns the issue of financial openness. Proofs of propositions are collected in appendix.

# 1. The basic model

In the basic model we consider a global economy that consists of two countries engaged in the bilateral exchange of goods for commodities. Country 1 is endowed with production capital and a manufacturing technology, and country 2 – with an oil stock and an oil extraction technology. The oil stock is renewable through investment in its maintenance and extension. Oil is absent in country 1 and manufacturing is absent in country 2.

Bilateral trade is essential for the global economy. Oil is exchanged for the final good on the competitive international markets under complete specialization of the economies. All oil extracted in country 2 is exported to country 1 and used for the final good production as an intermediate energy input making the capital workable. The final good is used for consumption and investment in both countries and also as an intermediate input in oil extraction.

Each household in country 1 owns a final good-producing firm and in country 2 – an oil-extracting firm. Firms of each type are homogenous among households. The latter make decisions on consumption and investment in the production factors under their ownership.

## 1.1. A two-country world

Countries differ in the size of population which is assumed constant. The number of households is $l_1$ in country 1 and $l_2$ in country 2, and the total world population is $l_1 + l_2 = 1$. Households are representative agents with homogenous preferences living indefinitely and maximizing the integral discounted utilities of consumption.

The country 1 household problem is to maximize

$$v_1 = \sum_{t=0}^{\infty} \beta^t u(c_{1t}), \tag{1.1}$$

subject to a sequence of budget constraints per period

$$c_{1t} + i_{1t} = y_{1t}, \tag{1.2}$$

and a sequence of equations for production capital accumulation:

$$k_{t+1} = (1-d)k_t + i_{1t}, \tag{1.3}$$

where $c_{1t}$ is the household consumption in period *t*, $u(c_{1t})$ is per-period utility, $\beta \in (0,1)$ is the discount factor, $y_{1t}$ is the country 1 household income, $k_t$ is physical capital per capita, $i_{1t}$ is investment in capital, $d$ is the rate of capital depreciation. According to (1.2), the household income per period is divided between consumption and investment, and to (1.3), a unity of investment in capital is transformed into a unity of new capital equipment.

A household in country 1 is endowed with an initial capital stock $k_0$ and a technology for the final good production. The output per firm owned by a household is a function of an oil-backed capital, $f_t = f(\hat{k}_t)$, which is, in turn, a Leontieff function of physical capital and oil input:[4]

[4] This is an extreme assumption adopted here for the sake of simplicity and to emphasize complementarity of energy and capital pointed out in the introduction. A more realistic approach allows for *ex ante* substitutability of energy and capital for newly installed equipment. For instance, Wei (2003) utilizes a putty-clay production technology with energy consumption and heterogeneous capital vintages.

$$\hat{k}_t = \min(k_t, q_t / \theta) . \qquad (1.4)$$

Here $q_t$ is oil input into the final good production in period *t*, and $\theta$ is the parameter of oil input in relation to physical capital $k_t$. Production function $f(\hat{k}_t)$ is neoclassical and satisfying the Inada conditions.

The country 1 household is self-employed and supplies a unity of labor to its firm. The household income per period is equal to the final output, less the oil purchase from country 2 in per capita terms:

$$y_{1t} = f(\hat{k}_t) - P_t q_t . \qquad (1.5)$$

where $P_t$ is the oil price in units of the final good which is a numeraire. The amount of oil purchase $P_t q_t$ in (1.5) is equivalent to the number of final goods exchanged by the household for oil in trade with country 2.

The household problem for country 2 is to maximize

$$v_2 = \sum_{t=1}^{\infty} \beta^t u(c_{2t}) , \qquad (1.6)$$

subject to the per-period budget constraints

$$c_{2t} + i_{2t} = y_{2t} , \qquad (1.7)$$

and the dynamic equations for the oil stock owned by the household:

$$s_{t+1} = s_t - x_t + i_{2t} / \delta , \qquad (1.8)$$

where $c_{2t}$ is consumption by the country 2 household in period *t*, $s_t$ is the oil stock at the beginning of period *t*, $y_{2t}$ is the household income which is equal to the oil rent per field, $x_t$ is the amount of oil extracted from the field in period *t*, $i_{2t}$ is investment of the final good in the oil stock, $\delta$ is a coefficient indicating investment required for an incremental oil stock increase.[5] According to (1.7), the household income in country 2 is divided between consumption and investment in the oil stock. According to (1.8), investment provides maintenance of the oil stock compensating oil extraction $x_t$ in period $t$ and its extension by amount $s_{t+1} - s_t$ by the beginning of period $t+1$. A household is endowed initially with the oil stock $s_0$ that defines the initial field's size.

An oil field is identical to a competitive oil-producing firm. The oil rent per field is equal to the oil revenue less the extraction cost:

[5] The model would capture oil depletion if the productivity of investment in oil stock is assumed to be decreasing with cumulated volume of oil already extracted indicating scarcity: $\delta_t = \delta(\sum_{\tau=0}^{t} x_\tau)$, $\delta_t' > 0$.

$$y_{2t} = P_t x_t - z_t, \quad (1.9)$$

where $z_t$ is the final good input in oil extraction. The oil extraction technology of the field is given by a homogenous of degree one, twice continuously differentiable production function of the oil stock and the intermediate input: $x_t = \Phi(s_t, z_t)$.[6] *An extraction rate* is the amount of oil extracted during one period in relation to the stock. It is a monotonously increasing and strictly concave function of the intermediate input intensity:

$$x_t / s_t = \phi(z_t / s_t) = \Phi(1, z_t / s_t), \quad (1.10)$$

satisfying the feasibility constraint $\phi(z_t / s_t) < 1$ and the Inada conditions: $\phi'(0) = \infty$, $\phi'(\infty) = 0$, and having the shape depicted in figure 1.[7] Under these assumptions the extraction is positive for any oil price and the oil stock cannot be exhausted at any period.

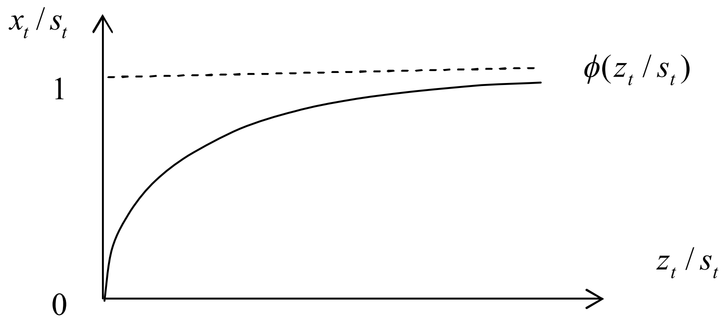


**Figure 1: The extraction rate function**

Global equilibrium under financial autarky is a sequence of households' plans for the final good production and oil extraction $(f_t, q_t, x_t)$, consumption-investment $(c_{jt}, i_{jt}), j = 1,2$, and accumulation of production assets $(k_t, s_t)$ solving the households' problems for country 1, (1.1)-(1.5), and country 2, (1.6)-(1.9), and satisfying at any period the oil market-clearing condition:

$$Q_t = X_t. \quad (1.11)$$

where $Q_t = l_1 q_t$ is the total demand for oil and $X_t = l_2 x_t$ is the total supply of oil.

[6] We thus ignore labor input, which is insignificant in the oil industry, and longevity of oil extraction equipment. By the oil stock we mean proven or exploited reserves.

[7] An example of such function is $\phi(z_t / s_t) = \dfrac{(z_t / s_t)^\alpha}{1 + (z_t / s_t)^\alpha}$, $0 < \alpha < 1$.

The one-period household budget constraints (1.2), (1.7) and the oil market equilibrium (1.11) imply that the market-clearing condition for the final good is fulfilled as identity:

$$F_t \equiv C_{1t} + C_{2t} + I_{1t} + I_{2t} + Z_t . \qquad (1.12)$$

where $F_t = l_1 f_t$ is total output by country 1, $C_{1t} = l_1 c_{1t}$ $C_{2t} = l_2 c_{2t}$ is total consumption in country 1 and 2, respectively, $I_{1t} = l_1 i_{1t}$, $I_{2t} = l_2 i_{2t}$ is total investment in the capital stock in country 1 and in the oil stock in country 2, respectively, $Z_t = l_2 z_t$ is total amount of the final good used as the intermediate input in oil extraction.[8]

## *1.2 Production and investment decisions*

The decision on oil extraction is adopted in any period by a country 2 household to maximize the one-shot consumption utility[9] $u(c_{2t}) = u(P_t x_t - z(x_t, s_t) - \delta(s_{t+1} - s_t + x_t))$, where $z(x_t, s_t) = \phi^{-1}(x_t / s_t) s_t$ is the extraction cost function. The first-order condition for $x_t$ implies that the oil price is the sum of the marginal extraction cost and the marginal cost of oil stock maintenance compensating for oil extraction in the field in the current period:

$$P_t = p_t + \delta \qquad (1.13)$$

where $p_t = \partial z / \partial x_t$ is the marginal extraction cost. The equilibrium oil price (1.13) is the sum of the "static" term $p_t$ related to oil extraction in the current period and the "dynamic" term $\delta$ related to the oil stock restoring by the beginning of the next period. The marginal extraction cost $p_t$ differs from the oil price $P_t$ by constant term $\delta$, and for the sake of briefness the term $p_t$ will be called in what follows the oil price, as well as $P_t$, whenever it does not lead to confusion.[10]

The oil supply per field is $x_t = s_t \xi(p_t)$, where $\xi(p_t)$ is the equilibrium extraction rate as an increasing function of the oil price: $\xi(p_t) = \phi(\zeta(p_t^{-1}))$, $\zeta = \phi'^{-1} = z_t / s_t$ is the equilibrium intensity of intermediate input. The function $\xi(p_t)$ satisfies: $\xi(0) = 0$,

[8] The final good output by country 1, $F_t$, differs from the global GDP, $Y_t$, which is the sum of GDPs across countries: $Y_t = Y_{1t} + Y_{2t} = C_{1t} + C_{2t} + I_{1t} + I_{2t}$, where $Y_{1t} = l_1 y_{1t}$, $Y_{2t} = l_2 y_{2t}$.

[9] This results from combining the household budget constraint per period (1.7), the oil stock accumulation equation (1.8), and the oil rent equation (1.9).

[10] Under the model extension pointed out in footnote 5 and capturing oil depletion, the oil price equation (1.13) would transform as: $P_t = p_t + \delta_t + \delta_t' i_{2t} / \delta_t$. The second term is related to the oil stock maintenance and is increasing in time since $\delta_t' > 0$. The third term compensates for an increase of the cost of oil stock extension.

$\lim_{p_t \to \infty} \xi(p_t) = 1$, $\lim_{p_t \to \infty} \xi'(p_t) = 0$. The oil extraction function $\phi(z_t / s_t)$ is assumed to have a sufficiently high curvature to guarantee that the equilibrium extraction rate $\xi(p_t)$ is strictly concave. This is the case for $\alpha < 1/2$ in the example of oil extraction technology given in footnote 7.

The oil rent per field is

$$y_{2t} = s_t(\delta\xi(p_t) + r(p_t)), \tag{1.14}$$

where $r(p_t) = p_t\xi(p_t) - \phi^{-1}(\xi(p_t))$ is monotonously increasing, strictly convex, and $r(0) = r'(0) = 0$. The marginal oil rent per barrel of oil in the ground is $\delta\xi(p_t) + r(p_t)$. It is the sum of the "dynamic" marginal rent $\delta\xi(p_t)$ compensating for an incremental decrease of the oil stock proportionally to the extraction rate, and of the "static" marginal rent $r(p_t)$ rewarding the owner of the oil field. The term $\phi^{-1}(\xi(p_t))$ is the extraction cost per barrel of oil in the ground as a function of the equilibrium extraction rate. The latter is linked with the static marginal oil rent as $\xi(p_t) = r'(p_t)$, according to Hotelling Lemma. Though $r'(p_t)$ converges to 1 as $p_t$ tends to infinity (since $\xi(p_t)$ converges to 1), the function $r(p_t)$, depicted in figure 2, has no asymptote line.[11]

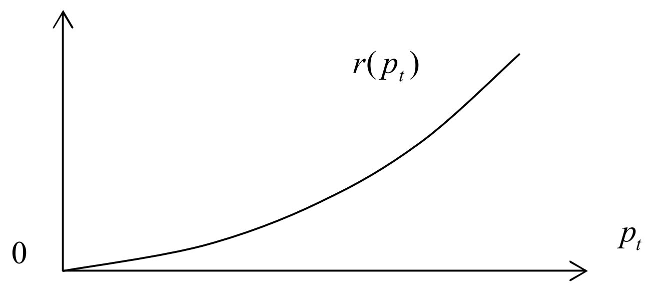


**Figure 2: The static marginal oil rent**

The final output per capita is $f(\hat{k}_t) = f(k_t)$, given that the production inputs are balanced: $k_t = q_t/\theta$. The oil demand by a country 1 household is $q_t = \theta k_t$. The oil market clears if $\theta k_t l_1 = s_t l_2 \xi(p_t)$, and the equilibrium oil price is

[11] $\lim_{p\to\infty}(r(p) - p) = \lim_{p\to\infty}[p(\xi(p) - 1) - \phi^{-1}(\xi(p)] = -\infty$ since $0 \le \lim_{p\to\infty} p(1 - \xi(p)) \le \infty$ and $\lim_{p\to\infty} \phi^{-1}(\xi(p)) = \infty$.

$$p_t = \varphi(\theta\kappa_t), \qquad (1.15)$$

where $\varphi = \xi^{-1}$ is the inverse of the equilibrium extraction rate, and $\kappa_t = \lambda k_t / s_t$ is *the global factor structure* (the capital-to-oil stock ratio), where $\lambda = l_1 / l_2$ is the relative population size of country 1. The function $\varphi$ is defined on the unit interval $[0,1)$, monotonously increasing, strictly convex, and $\varphi(0) = 0$, $\varphi(1) = \infty$.

The country 1 household income is $y_{1t} = f(k_t) - P_t\theta k_t = f(k_t) - \delta\theta k_t - \theta p_t k_t$, according to (1.13). Taking into account the household budget constraint (1.2), the capital accumulation equation (1.3) is represented as

$$k_{t+1} + c_{1t} = f(k_t) + (1 - D - \theta p_t)k_t \qquad (1.16)$$

where $D = d + \delta\theta$. The marginal return on capital in period $t$ is, thus, $R_t^k = 1 + f'(k_t) - D - \theta p_t$. According to (1.16), the marginal return on capital equals to the gross marginal product of capital, $1 + f'(k_t)$, less the physical depreciation rate $d$ and the oil purchase per unit of capital $\theta P_t = \theta(\delta + p_t)$. The term $D$ in (1.16) can be called a *generalized* rate of capital depreciation to emphasize that the country 1 household income is the financial source for both the capital replacement in country 1 and the oil stock maintenance in country 2. The oil stock "depreciates" at the rate of oil extraction $\xi(p_t)$ which is covered by term $\delta\theta$ of the generalized depreciation rate $D$.

Combining the oil stock accumulation equation (1.8) with the country 2 household budget constraint (1.7), the oil supply, and the oil rent (1.14) yields $c_{2t} = y_{2t} - \delta(s_{t+1} - s_t + x_t)$ $= \delta\xi(p_t)s_t + r(p_t)s_t - \delta(s_{t+1} - s_t + \xi(p_t)s_t)$ or

$$\hat{s}_{t+1} + c_{2t} = (1 + r(p_t)/\delta)\hat{s}_t \qquad (1.17)$$

where $\hat{s}_t = \delta s_t$ denotes the oil stock measured in the final good unit.[12] The marginal return to this stock is $R_t^s = 1 + r(p_t)/\delta$. The returns $R_t^k$ and $R_t^s$ govern the intertemporal choices by households in both countries and do not, generally, coincide under financial autarky.

*Proposition 1. The equilibrium Euler equations are*

$$u'(c_{1t-1}) = \beta R_t^k u'(c_{1t}), \qquad (1.18)$$

$$u'(c_{2t-1}) = \beta R_t^s u'(c_{2t}). \qquad (1.19)$$

[12] In equilibrium the in-ground value of oil in the final good unit is equal to $\delta$ and is therefore constant in our model.

An equilibrium path of the global economy is given by the bundle ($p_t, k_t, s_t, c_{jt}, j = 1,2$) satisfying the equations for the market-clearing oil price (1.15), the production factors accumulation (1.16), (1.17), and the intertemporal choice (1.18), (1.19). This path is determined by the initial endowments of capital and oil stocks and the initial consumption choice satisfying the intertemporal budget constraints for each economy.[13] It is important to emphasize that the equilibrium path under financial autarky relies on simultaneous consumption-investment decisions by households in each country.

## 1.3 The steady-state equilibrium

Consider a stationary equilibrium path along which all variables are constant in time. Equating consumption over time, $c_{jt-1} = c_{jt}, j = 1,2$, yields the Euler equations for this path (1.18), (1.19) implying that $\beta R_t^k = \beta R_t^s = 1$ or

$$f'(k) = \rho + D + \theta p, \tag{1.20}$$

$$r(p)/\delta = \rho, \tag{1.21}$$

where $\rho = \beta^{-1} - 1$ is the household discount rate. The time subscript is omitted in the notation of steady-state variables. Equation (1.20) defines a positive link between the marginal product of capital and the oil price. It extends the golden rule of capital accumulation, $f'(k_{gold}) = \rho + d$. The higher is oil input coefficient $\theta$, the more significant is deviation from this rule, as figure 3 illustrates.

[13] Applying the dynamic programming would yield consumption functions as $c_{1t} = c_1(k_t, p_t)$, $c_{2t} = c_2(R_t^s s_t)$ and reduce a five-dimensional dynamic system for the global economy (1.15)-(1.19) to two difference equations on $k_t$ and $s_t$ (since $p_t$ is defined by the global factor structure).

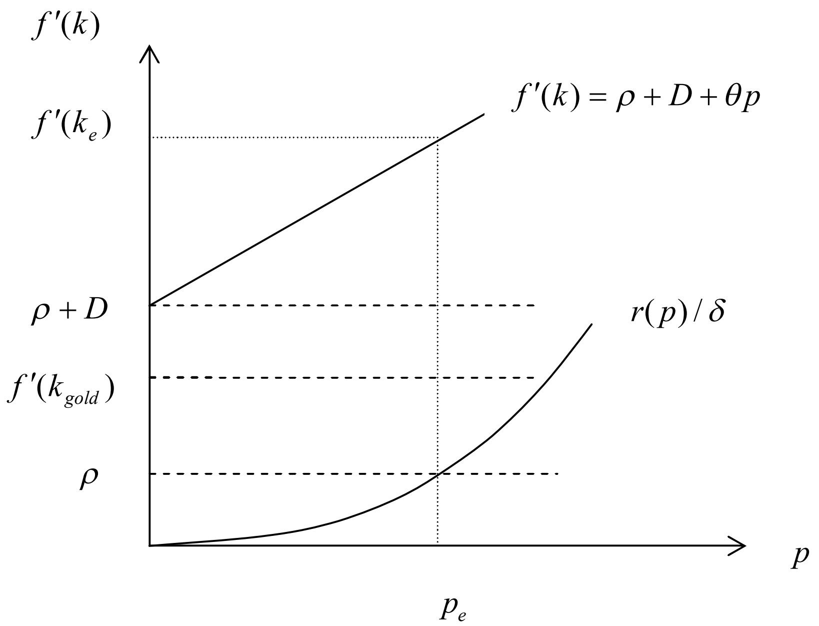


**Figure 3: The equilibrium steady-state**

Equation (1.21) links the marginal static oil rent with the household discount rate. Its solution, the stationary oil price $p_e$ is high (the subscript *e* relates to the equilibrium steady-state) if households are impatient ($\rho$ is high) or investment in oil stocks are low-productive ($\delta$ is high). Figure 3 shows the steady-state equilibrium defined by the pair $(p_e, k_e)$, where $f'(k_e)=\rho+D+\theta p_e$ is the steady-state marginal product of capital. From the budget constraints (1.16), (1.17), the steady-state consumption is[14]

$$c_{1e}=f(k_e)-(D+\theta p_e)k_e \tag{1.22}$$

$$c_{2e}=\theta\lambda k_e r(p_e)/\xi(p_e) \tag{1.23}$$

Consumption in both countries positively relates to capital $k_e$ which is decreasing in the oil price $p_e$. Consumption in country 1 depends negatively on the oil price, while for country 2 this dependence is ambiguous. According to (1.23), all static oil rent is consumed fully by country 2, because maintenance of the steady-state oil stock is financed implicitly by country 1 through oil price $P_e$.

[14] The steady-state capital stock $k_e=f'^{-1}(\rho+D+\theta p_e)$ is decreasing in the oil price. The steady-state oil stock is calculated from the oil price equation (1.15) as $s_e=\theta\lambda k_e/\xi(p_e)$. The steady-state investments are found from the factor accumulation equations (1.3), (1.8) as $i_{1e}=dk_e$, $i_{2e}=\delta x_e=\delta\theta k_e$.

# 2. The global planner problem

Consider a global planner maximizing the weighted integral utility: $v^{\mu} = \mu l_1 v_1 + l_2 v_2$, where $\mu > 0$ is a relative weight placed on country 1. The individual utility is also weighted with the population size of countries $l_j, j = 1,2$. At any period the planner makes a decision on investment-consumption, production, allocation of goods, and distribution of global income between the countries. The set of control variables include the oil price $P_t$ and a lump-sum transfer of the final good $T_t$ from country 1 to country 2.

## 2.1. The optimal model

The problem is to maximize the integral weighted utility

$$v^{\mu} = \sum_{t=0}^{\infty} \beta^t \left( \mu l_1 u(c_{1t}) + l_2 u(c_{2t}) \right), \qquad (2.1)$$

subject to the condition of the final good distribution:

$$F_t = C_{1t} + C_{2t} + I_{1t} + I_{2t} + Z_t, \qquad (2.2)$$

the resource constraint for oil:

$$\theta K_t \le X_t, \qquad (2.3)$$

the budget constraint per period for country 1:

$$C_{1t} + I_{1t} = Y_{1t}, \qquad (2.4)$$

and the equations for capital and oil stocks accumulation:

$$K_{t+1} = (1-d)K_t + I_{1t}, \qquad (2.5)$$

$$S_{t+1} = S_t - X_t + I_{2t} / \delta . \qquad (2.6)$$

where $K_t = l_1 k_t$ and $S_t = l_2 s_t$ is the total stock of capital and oil, respectively, $Y_{1t} = y_{1t} l_1 = F_t - P_t X_t - T_t$, and $T_t$ is the final good transfer from economy 1 to 2 if $T_t \ge 0$, and the other way round if $T_t < 0$.

Equation (2.2) is fulfilled as identity (1.13) in the equilibrium model. The resource constraint (2.3) is implied from the Leontieff production function (1.5) represented on the aggregate level as $\hat{K}_t = \min(K_t, Q_t / \theta)$. From (2.2) and (2.4), the budget constraint for country 2 holds as identity: $C_{2t} + I_{2t} \equiv Y_{2t}$, where $Y_{2t} = y_{2t} l_2 = P_t X_t + T_t - Z_t$. Given the initial world stocks of capital and oil, $K_0$ and $S_0$, the planner chooses at any time period the oil price $P_t$, the transfer $T_t$, the final good output $F_t$, the intermediate input $Z_t$, the oil extraction $X_t$, and the consumption-investment bundle $C_{jt}$, $I_{jt}$ ($j = 1,2$) solving the problem (2.1)-(2.6).

### 2.2 Optimal oil price and investment

The oil price and the transfer are the instruments of the world income distribution. It is shown in appendix that the optimal oil price ensures equalization of the weighted marginal household utilities:[15]

$$\mu u'(c_{1t}) = u'(c_{2t}) . \tag{2.7}$$

The higher is the relative weight of country 1, the larger is consumption per capita of this country relative to country 2. As is also shown in appendix, optimal oil price is the sum of the marginal costs of oil extraction and oil stock maintenance for any field, $P_t = p_t + \delta$ . The optimal oil extraction and the oil rent per field coincide with the equilibrium ones: $x_t = s_t \xi(p_t)$, $y_{2t} = s_t(\delta\xi(p_t) + r(p_t))$. The equality of total oil supply and demand, $S_t \xi(p_t) = \theta K_t$, yields the oil price schedule the same as (1.15), $p_t = \varphi(\theta\kappa_t)$.

The equations for accumulation of factors are

$$K_{t+1} + C_{1t} = F(K_t) + (1 - D - \theta p_t)K_t - T_t , \tag{2.8}$$

$$\hat{S}_{t+1} + C_{2t} = (1 + r(p_t)/\delta)\hat{S}_t + T_t , \tag{2.9}$$

where $\hat{S}_t = \delta S_t$ is the total oil stock measured in the final good units. In making investment decisions, the planner takes into account that the oil price $p_t$ depends on the global factor structure $\kappa_t$ predetermined by investment choice of the previous period. The right-hand sides of (2.8), (2.9) depend on this price, and the total effects of investment on consumption are captured by the marginal aggregate consumption rates with respect to the production factors, $\partial C_{jt} / \partial K_t$, $\partial C_{jt} / \partial \hat{S}_t$, $j = 1,2$ .

*Proposition 2. The marginal aggregate consumption rates for the global planner's problem are*

$$\partial C_{1t} / \partial K_t = R_t^k - \Delta R_t^k , \quad \partial C_{2t} / \partial K_t = \Delta R_t^k , \tag{2.10}$$

$$\partial C_{1t} / \partial \hat{S}_t = \Delta R_t^s , \quad \partial C_{2t} / \partial \hat{S}_t = R_t^s - \Delta R_t^s , \tag{2.11}$$

*where* $\Delta R_t^k = \theta^2 \varphi'(\theta\kappa_t)\kappa_t , \Delta R_t^s = \theta\xi(p_t)\varphi'(\theta\kappa_t)\kappa_t / \delta$ .

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[15] The same first-order condition is obtained for the optimal transfer, but this does not mean redundancy of this control variable, as will be clear further below.

The marginal consumption rate of country 1 with respect to capital, $\partial C_{1t} / \partial K_t$, is equal to the return on capital $R_t^k$, less the marginal effect of capital growth $\Delta R_t^k$ causing an increase of the oil price and transferred to country 2. Similarly, the marginal consumption rate $\partial C_{2t} / \partial \widehat{S}_t$ is equal to the oil stock return $R_t^s$, less the marginal effect of the oil stock extension $\Delta R_t^s$ causing a decrease of the oil price and transferred to country 1. The planner has to internalize these distribution effects, ignored by households. But being captured through the optimal oil price ensuring (2.7), these effects can be neglected in optimal investment decision, as the next proposition demonstrates.

*Proposition 3. The optimal consumption-investment path satisfies the Euler equations:*

$$u'(c_{1t-1}) = \beta R_t^k u'(c_{1t}), \qquad (2.12)$$

$$u'(c_{2t-1}) = \beta R_t^s u'(c_{2t}). \qquad (2.13)$$

Though these equations coincide with (1.19), (1.20), the optimal and equilibrium paths are, generally, not identical because the returns $R_t^k$ and $R_t^s$ are determined differently in the optimal and equilibrium models. The optimal oil price condition (2.7) implies that the marginal utilities ratio $u'(c_{1t}) / u'(c_{2t})$ is constant in time and, hence, the marginal returns on capital and the oil stock are equalized at any period, $R_t^k = R_t^s$.

This requirement imposes a condition on the optimal oil price:

$$r(p_t) / \delta + \theta p_t = f'(k_t) - D, \qquad (2.14)$$

Since the left-hand side of this equation is monotonously increasing in $p_t$, it has one positive root $p_t^*$, given that $f'(k_t) > D$ (the asterisk refers to optimal solution). From (2.14), the optimal oil price is a decreasing function of capital: $p_t^* = p(k_t)$, $p'(k_t) < 0$, and it is also decreasing in $\theta$. The reason for this is that any increase of oil demand leads to a decrease of $R_t^k$ and an increase of $R_t^s$. To cancel these effects out, the optimal oil price decreases exerting a "stabilizing" comparative static effect on the returns.

Inserting $p(k_t)$ into the country 1 budget constraint (2.8) written as $c_{1t} = f(k_t) + (1 - D - \theta p(k_t))k_t - \tau_{1t} - k_{t+1}$, where $\tau_{1t} = T_t / l_1$ is transfer per capita, and inserting the optimal return on capital $R^k(k_t) = 1 + f'(k_t) - D - \theta p(k_t)$ into the Euler equation (2.12), yields a dynamic system for $c_{1t}, k_t$. It defines country 1 optimal path, given the optimal transfer path $T_t$, irrespective of country 2 dynamic. The latter is predetermined by the capital path in the following way.

The returns-equalizing oil price $p(k_t)$ must coincide with the market-clearing oil price $p_t = \varphi(\theta\kappa_t)$. This is fulfilled if $\theta\kappa_t = \varphi^{-1}(p(k_t)) = \xi(p(k_t))$, implying that the optimal oil stock is

$$s(k_t) = \theta\lambda k_t / \xi(p(k_t)). \tag{2.15}$$

The optimal oil stock is thus an increasing and convex function of capital since $\xi'(p_t) > 0$, $p'(k_t) < 0$. Due to (2.15), the volume of investment required to ensure the optimal oil stock $\hat{s}(k_{t+1})$ is $i_{2t}^* = \hat{s}(k_{t+1}) - \hat{s}(k_t) + \delta\lambda\theta k_t$. Consumption in country 2 should, hence, satisfy the budget constraint (2.9) represented as $c_{2t} = (1 + r(p(k_t))/\delta)\hat{s}(k_t) + \tau_{2t} - \hat{s}(k_{t+1})$, $\tau_{2t} = T_t / l_2$, and the Euler equation (2.13) with $p_t = p(k_t)$. These two equations for optimal consumption path of country 2 are compatible due to the choice of transfer path. Investment decisions are thus governed by the optimal oil price and are adopted in the sequential fashion at any period, unlike the equilibrium model with simultaneous choices by the countries.

Given the initial capital stock, $k_0$, the optimal path of country 1 is selected through the choice of $c_{10}^*$ satisfying the intertemporal budget constraint for this country (obtained by iterating (2.8) over time):

$$\sum_{t=0}^{\infty} \left(\Pi_{i=0}^{t} R^k(k_i)\right)^{-1} (c_{1t} + i_{1t} + \tau_{1t} - y_{1t}) = 0. \tag{2.16}$$

The initial consumption for country 2 is found from (2.7) as $c_{20}^* = u'^{-1}\left(\mu^{-1} u'(c_{10}^*)\right)$ or $c_{20}^* = \mu^{-1/\sigma} c_{10}^*$ for the case of isoelastic utility $u(c) = (c^{1-\sigma} - 1)/(\sigma - 1)$, $\sigma > 0$. This relationship should be valid for the initial oil stock $s_0$ and any relative utility weight $\mu$. The latter is matched with the time-averaged expected transfer $\overline{T}$ derived from the intertemporal budget constraint for country 2 similar to (2.16): $\overline{T} = \sum_{t=0}^{\infty} \vartheta_t T_t$, where $\vartheta_t = \sum_{\tau=0}^{t} \left(\Pi_{i=0}^{\tau} R^k(k_i)\right)^{-1} / \sum_{\tau=0}^{\infty} \left(\Pi_{i=0}^{\tau} R^k(k_i)\right)^{-1}$ is the time weight of period-$t$ transfer.

## *2.3 The steady state*

Along the stationary optimal path the Euler equation (2.12) for country 1 transforms into $\beta R^k(k) = 1$ or $f'(k) = \rho + D + \theta p(k)$. The stationary optimal oil price coincides with the stationary equilibrium oil price,[16] $p(k)=p_e$ implying that the optimal capital stock is the same as in the steady-state equilibrium, $k^*=k_e$. The optimal oil stock is also the same, $s^*=s_e=\theta\lambda k_e/\xi(p_e)$, as well as investments in these stocks: $i_1^*=dk_e$, $i_2^*=\delta\theta k_e$.

Optimal consumption, however, differs from the equilibrium one due to the transfer:

$$c_1^*=f(k_e)-(D+\theta p_e)k_e-\tau_1^* \tag{2.17}$$

$$c_2^*=\theta\lambda k_e r(p_e)/\xi(p_e)+\tau_2^* \tag{2.18}$$

where $\tau_j^*=T^*/l_j$ is the stationary optimal transfer per capita, $j=1,2$. It is calculated for the isoelastic utility from the marginal utility equalization (2.7) implying $c_1^*=\mu^{1/\sigma}c_2^*$ for isoelastic utility. Combining this with (2.17), (2.18) and rearranging terms yields $T^*=\dfrac{f(k_e)-\left(D+\theta(p_e+\mu^{1/\sigma}\lambda r(p_e)/\xi(p_e))\right)k_e}{l_1^{-1}+\mu^{1/\sigma}l_2^{-1}}$. A higher utility weight of country 1 implies a lower transfer to country 2.

As a result, the equilibrium and optimal stationary paths coincide for all variables but consumption because of the equalization of equilibrium factor returns in the steady state. Optimal consumption is shifted from the equilibrium one due to the global income redistribution through optimal transfer corresponding to utility weight $\mu$. The equilibrium steady-state path is optimal for a special weight corresponding to the zero transfer, $T=0$.

### *2.4 The international bond market*

Since the Euler equations coincide for the equilibrium and optimal models, the planner can rely on individual choices of consumption and investment, provided that the returns on production factors are equalized at any period.[17] The financial autarky equilibrium does not ensure such equalization (except for the stationary path). The simplest extension of the model to financial openness is given by introduction of the international bond market. Let households in both countries trade in one-period bonds with risk-free return $R_t$ and let $b_{jt}$ denote the foreign bond holding by a country $j$ household. The household budget constraints

[16] Inserting $f'(k)$ into the optimal oil price equation (2.14), yields $r(p(k))/\delta+\theta p(k)-(f'(k)-D)=$ $r(p(k))/\delta+\theta p(k)-(\rho+\theta p(k))=0$ or $r(p(k))/\delta=\rho$ which is equivalent to the equilibrium steady-state Euler equation (1.21).

[17] The effect of investment on the oil price and the factor returns is neutralized in the optimal model by the redistribution effect of the marginal utilities equalization. Individuals make investment decisions as if they capture these effects by neglecting both.

are rewritten as $k_{t+1} + b_{1t+1} + c_{1t} = f(k_t) + (1 - D - \theta p_t)k_t + R_t b_{1t}$ for country 1, and $\widehat{s}_{t+1} + b_{2t+1} + c_{2t} = R_t^s \widehat{s}_t + R_t b_{2t}$ for country 2.

The bond market is in equilibrium, $\lambda b_{1t} + b_{2t} = 0$, only if the returns on bonds and production factors are equalized: $R_t = R_t^k = R_t^s$. The second equality is fulfilled for the returns-equalizing oil price $p(k_t)$ determined in period $t-1$ simultaneously with $R_t^k$ and $R_t^s$. One can interpret this price as a forward oil price $p_t^f = p(k_t)$ contracted implicitly in period $t-1$ to provide the equalization of factor returns and coinciding with the spot price that clears the market in period $t$, $p_t^f = p_t$. Country 2 households infer from this equation that the oil stock in this period depends on capital according to the optimal oil stock condition (2.15). They make consumption-investment decisions basing on the capital-linked investment schedule $i^*_{2t-1}$ and by obeying the budget constraint (1.7) and the Euler equation (1.19). These conditions are compatible due to the new bonds issues or purchases, $b_{2t} - R_t b_{2t-1}$, similarly to optimal transfer $\tau_{2t-1}$ in the planner's model.

International trade in bonds thus ensures equalization of factor returns and optimality of equilibrium path. The initial debt per capita $b_{j0}$ corresponds to utility weight $\mu$ in the same way as the expected time-average transfer $\overline{T}$ corresponds to some weight in the planner's model. For example, the higher is the initial external debt of country 1 measured by $b_{20}$, the larger is the time-average transfer from country 1 to 2, and the lower is the utility weight of country 1.

## 3. A model with uncertainty

The basic equilibrium model is extended in this section to consider investment under uncertainty and the issues of risk sharing between the economies. It is assumed that the technological parameter of oil input per unit of capital, $\theta_t$, is time-varying and stochastic. Fluctuations of this parameter affect the global economy through the uncertain oil demand $\theta_t K_t$ and the oil price $P_t(\theta_t)$.

The stochastic process for $\theta_t$ is assumed to be driven by a Markov chain. Let $N$ be the number of states of nature and $\theta^i$, $i = 1,...,N$, be realizations of $\theta_t$ in each state ordered as $\theta^1 < \theta^2 < ... < \theta^N$. The unconditional probability distribution in period $t = 0,1,...$ is $\pi_t^i$, the transition probability is $\pi(\theta_t|\theta_{t-1}) = \Pr(\theta_t = \theta^g|\theta_{t-1} = \theta^i) \equiv \pi^{ig}$, $\sum_{g=1}^{n}\pi^{ig} = 1$, and the law of motion for the probability distribution over states is $\pi_t^g = \sum_{i=1}^{n}\pi^{ig}\pi_{t-1}^i$ $(i, g = 1,...,N)$.

### 3.1 Stochastic equilibrium under financial autarky

Let households maximize the expected discounted utilities of consumption stream, $v_j = E_0\sum_{t=0}^{\infty}\beta^t u(c_{jt})$, $j = 1,2$, where $E_0$ is the mathematical expectation conditional on time-0 information, subject to the budget constraints and the capital/oil stock accumulation equations that are the same as in the basic model of section 1.1. Production and trade take place in each period $t$ after the realization of $\theta_t = \theta^i$ has been observed, but investments in production factors are decided in period $t-1$, prior to the resolution of uncertainty. Though the production factors $k_t$ and $s_t$, as well as the final output $f(k_t)$, are known with certainty by the beginning of period $t$, the global income distribution is uncertain because the equilibrium oil price $P_t(\theta_t)$ is unknown.

The static equilibrium conditions derived above for the deterministic model are valid for any $\theta_t$ observed in period $t$: the equilibrium oil price is $P_t(\theta_t) = \delta + p_t(\theta_t)$ where $p_t(\theta_t) = \varphi(\theta_t\kappa_t)$ is the marginal extraction cost term as the function of the oil input coefficient times the factor structure. The equilibrium household incomes are $y_{1t}(\theta_t) = f(k_t) - dk_t - (\delta + p_t(\theta_t))\theta_t k_t$, $y_{2t}(\theta_t) = s_t(\delta\xi(p_t(\theta_t)) + r(p_t(\theta_t)))$. The capital return $R_t^k(\theta_t) = 1 + f'(k_t) - D(\theta_t) - p_t(\theta_t)\theta_t$, $D(\theta_t) = d + \delta\theta_t$, is negatively correlated with the oil demand shocks $\theta_t$, while the oil stock return $R_t^s(\theta_t) = 1 + r(p_t(\theta_t))/\delta$ is positively correlated with these shocks. The stochastic equilibrium Euler equations are

$$u'(c_{1t-1}) = \beta E_{t-1}R_t^k u'(c_{1t}), \quad (3.1)$$

$$u'(c_{2t-1}) = \beta E_{t-1}R_t^s u'(c_{2t}). \quad (3.2)$$

where $E_{t-1}R_t^l u'(c_{jt}) = \sum_{g=1}^{n}R_t^l(\theta^g)u'(c_{jt}(\theta^g))\pi^{ig}$, given that $\theta_{t-1} = \theta^i$, is the conditional expectation built on information of date $t-1$ and the Markov transition probability ($l = k, j = 1$ or $l = s, j = 2$).

Under stationary stochastic equilibrium, consumption in country $j = 1,2$ and production factors are contingent on the state of nature at any date $t$: $c_{je}(\theta^i) = c_{je}^i$,

$k_e(\theta^i) = k_e^i$, $s_e(\theta^i) = s_e^i$ if $\theta_t = \theta^i$.[18] This equilibrium is in general inefficient under financial autarky since the marginal rates of substitution $u'(c_{je}^g)/u'(c_{je}^i)$ are not equalized across states and countries. As we have seen, the stationary equilibrium is efficient in the deterministic case under financial autarky due to the long-run equalization of returns: $R^k = R^s = \beta^{-1}$. In the stochastic case there is no possibility for equalization of marginal returns across states unless the investment risks are traded between the countries.

### 3.2 The planner's problem

The planner maximizes the expected weighted integral utility of the countries $v^\mu = E_0 \sum_{t=0}^{\infty} \beta^t \left(\mu l_1 u(c_{1t}) + l_2 u(c_{2t})\right)$ subject to the equations for the global income distribution (2.2), (2.4)-(2.6), and the resource constraint for oil (2.3) written as $\theta_t K_t \le X_t$. The global economy is subject to aggregate uncertainty since the oil demand shock $\theta_t$ affects input decisions that cannot be eliminated through allocation of goods in period $t$. Both the final good and the oil extracted are non-storable and no reserves of these goods can be created to smooth consumption across states of nature.

The planner uses the same policy tools as in the deterministic model including the oil price $P_t(\theta_t)$ and the lump-sum transfer $T_t(\theta_t)$ from country 1 to 2 decided after realization of $\theta_t$ has been observed. The condition of weighted marginal utilities equalization across countries (2.7) is, hence, fulfilled for any state of nature,

$$\mu u'(c_{1t}(\theta^i)) = u'(c_{2t}(\theta^i)), \tag{3.3}$$

$i = 1,...,N$. Once the uncertainty about $\theta_t$ is resolved, $\theta_t = \theta^i$, the static optimal plans of production and trade in goods are calculated in the same way as in the deterministic model. As above, the effects of investment on the next-period oil price should be neglected under cross-country equalization of marginal utilities (3.3). Let $\tilde{E}_{jt-1}$ denote a conditional expectation for country $j$ defined on *the risk-adjusted* probability distribution $\tilde{\pi}_{jt}^{ig} = \pi^{ig} u'(c_{jt}(\theta^g)) / E_{t-1} u'(c_{jt})$, given that $\theta_{t-1} = \theta^i$.

[18] The oil-augmented golden rule of capital accumulation (1.20) and the marginal oil rent equation (1.21) are generalized as $f'(k_e^i) = \rho + d + \sum_{g=1}^{N} \theta^g (\delta + \varphi(\theta^g \kappa_e^i)) \pi^{ig} \chi_1^{ig}$ and $\sum_{g=1}^{N} r(\varphi(\theta^g \kappa_e^i)) \pi^{ig} \chi_2^{ig} = \delta\rho$, respectively, where $\kappa_e^i = \lambda k_e^i / s_e^i$, $\chi_j^{ig} = u'(c_{je}^g)/u'(c_{je}^i)$.

*Proposition 4. (i) The optimal path satisfies the stochastic Euler equations*

$$u'(c_{jt-1}) = \beta E_{t-1} R_t^l u'(c_{jt}), \qquad (3.4)$$

*where* $l = k, j = 1$ *or* $l = s, j = 2$*. For this path: ii) the risk-adjusted transition probabilities are the same for both countries:*

$$\tilde{\pi}_{1t}^{ig} = \tilde{\pi}_{2t}^{ig} \equiv \tilde{\pi}_t^{ig}$$

*for* $i, g = 1,...,N$*, and iii) the risk-adjusted returns are equalized:*

$$\tilde{E}_{t-1} R_t^k = \tilde{E}_{t-1} R_t^s. \qquad (3.5)$$

Though the optimal and equilibrium Euler equations are the same, the optimal and equilibrium investment plans coincide only under equalization of expected returns adjusted for the risk attitude of households (3.5). The country subscript is omitted in the notation of conditional risk-adjusted expectation $\tilde{E}_{t-1}$ which is identical for both countries due to the equalization of risk-adjusted probabilities.

Equation (3.5) is represented as $\tilde{E}_{t-1}[r(p_t^*(\theta_t))/\delta + \theta_t p_t^*(\theta_t) + \theta_t \delta] = f'(k_t) - d$. Imposing the condition that the optimal oil price ensures market clearing, $p_t^*(\theta_t) = \varphi(\theta_t \kappa_t)$, we have:

$$\tilde{E}_{t-1}[r(\varphi(\theta_t \kappa_t))/\delta + \theta_t \varphi(\theta_t \kappa_t) + \theta_t \delta] = f'(k_t) - d, \qquad (3.6)$$

This is an equation for optimal factor structure $\kappa_t$ extending the optimal oil price equation (2.14). Its left-hand side is increasing in $\kappa_t$, implying the unique solution $\kappa_t^*$. Consider a second-order approximation for $\kappa_t^*$ by assuming that the extraction rate $\theta_t \kappa_t$ is small[19]. Suppose that the oil extraction function $\phi(z_t / s_t)$ is of sufficiently high curvature near zero to guarantee that $\xi'(0) = \infty$, $-\xi''(0)/\xi'(0)^3 = \varphi''(0) = 0$ (the latter condition is fulfilled for our reference example in footnote 7 for $\alpha < 1/3$).

*Proposition 5. Under a small extraction rate the second-order approximation for the optimal factor structure yields:*

$$\kappa_t^{*2} \approx 2\delta \frac{f'(k_t) - d - \delta \tilde{E}_{t-1}\theta_t}{\tilde{E}_{t-1}\theta_t^2}. \qquad (3.7)$$

The nominator in the right-hand side of (3.7) is the marginal product of capital in period *t*, net off the expected generalized depreciation rate. The denominator is the second-

[19] The assumption of small extraction rate fits the stylized facts about the global oil industry. Smith (2009, p. 153) points out that OPEC's installed production facilities are sufficient to extract 1.5 percent of its proved reserves per year, while non-OPEC producers have installed facilities sufficient to extract 5.6 of their proved reserves each year.

order moment of $\theta_t$ built on the risk-adjusted transition probability $\tilde{\pi}_t^{ig}$. A possible interpretation of (3.7) is that the expected net marginal product of capital is equal to the risk premium calculated as $\kappa_t^{*2}\tilde{E}_{t-1}\theta_t^2/2\delta$ and rewarding households in both countries for the adoption of the risky factor structure.

The optimal oil stock is equal to $s_t^* = \lambda k_t / \kappa_t^*$ or, from (3.7):

$$s_t^* = \lambda k_t \left( \frac{\tilde{E}_{t-1}\theta_t^2}{2\delta(f'(k_t) - d - \delta\tilde{E}_{t-1}\theta_t)} \right)^{1/2}, \tag{3.8}$$

It is strictly increasing and convex in capital entering explicitly the right-hand side of (3.8), and is decreasing in the conditional moments $\tilde{E}_{t-1}\theta_t$ and $\tilde{E}_{t-1}\theta_t^2$. By the Markov property of $\theta_t$, all information about this parameter available in period $t-1$ is contained in $\theta_{t-1}$. One can assume that the structure of the transition probability matrix $(\pi^{ig})^{N\times N}$ is close enough to diagonal to guarantee a strong positive autocorrelation for $\theta_t$ sufficient for both conditional moments $\tilde{E}_{t-1}\theta_t$ and $\tilde{E}_{t-1}\theta_t^2$ to be increasing functions of the observed $\theta_{t-1}$. Then, from (3.8), $s_t^*$ should be increasing in $\theta_{t-1}$.

This property implies a qualitative rule for optimal investment policy requiring that the optimal factor structure $\kappa_t^*$ should decrease in response to a positive oil demand shock. Optimal oil stock investment exerts, according to (3.8), a stabilizing effect on optimal oil price dynamic. Realization of a high $\theta_{t-1}$ causes an upward movement of the oil price in period $t-1$ but, according to (3.7), has a downward effect on the oil price in period $t$, $\varphi(\theta_t \kappa_t^*)$, through a decrease of the optimal factor structure.[20]

## *3.3 The international market for state-contingent securities*

Introduction of the international bond market is sufficient for the deterministic equilibrium path to be optimal. The equilibrium model with uncertainty is extended here to show that opening of a market for state-contingent claims provides realization of the stochastic optimal path. Consider an international market for one-period Arrow securities providing mutual insurance for two countries: in states with low $\theta^g$ country 1 is, typically, an insurer, while in states with high $\theta^g$ it is insured by securities purchased from country 2.

[20] As above, consumption plans of the countries are compatible due to the choice of transfers $\tau_{2t-1}(\theta_{t-1})$.

A security issued in period $t$ is a promise to pay a dollar in each state $g = 1,...,N$ in period $t+1$. Let $a^g_{jt+1}$ denote state-$g$ security purchased in period $t$ by a household from country $j = 1,2$ at price $\psi^{ig}_t$ under state $\theta_t = \theta^i$. The household budget constraints are

$$k_{t+1} + a_{1t+1} + c_{1t} = f(k_t) + (1 - D(\theta_t) - \theta_t p_t(\theta_t))k_t + a^i_{1t}, \quad (3.9)$$

$$\hat{s}_{t+1} + a_{2t+1} + c_{2t} = R^s_t(\theta_t)\hat{s}_t + a^i_{2t}, \quad (3.10)$$

where $a_{jt+1} = \sum_{g=1}^{N} \psi^{ig}_t a^g_{jt+1}$ is the total purchase of state-contingent securities by the household. The markets for these securities clear if $l_1 a^g_{1t+1} + l_2 a^g_{2t+1} = 0$ for $g = 1,...,N$.

*Proposition 6. Trade in state-contingent securities yields: i) equalization of the risk-adjusted transition probabilities across countries for any state:*

$$\tilde{\pi}^{ig}_{1t} = \tilde{\pi}^{ig}_{2t} \equiv \tilde{\pi}^{ig}_t, \quad (3.11)$$

*ii) equalization of the risk-adjusted returns:*

$$\tilde{E}_{t-1} R^k_t = \tilde{E}_{t-1} R^s_t = R_t, \quad (3.12)$$

*where* $R_t = 1/\sum_{g=1}^{N} \psi^{ig}_{t-1}$ *is the risk-free rate of return, and* $\psi^{ig}_{t-1} = \tilde{\pi}^{ig}_t / R_t$.

The risk-free return in (3.12) is yielded by a uniformly weighted portfolio of these securities. Proposition 6 implies that trade in state-contingent securities results in the same allocation of risks and investment choices as those under the optimal oil price setting. The returns on investment in production factors are equalized in the same way as in the planner's model, through the selection of the global factor structure. The latter coincides with $\kappa^*_t$ due to the coincidence of the first-order conditions for investment choices (3.5) and (3.12). The market-clearing oil price is optimal in any state $\theta_t = \theta^g$ of period $t$ since $p^*_t(\theta^g) = \varphi(\theta^g \kappa^*_t)$, and households in both countries make investment decisions resulting in the optimal factor structure.

The returns-equalizing oil price was interpreted for the deterministic model as the implicit forward price set one period ahead, $p^f_t = p_t(k_t)$. The similar interpretation can be given for the bundle of state-contingent oil prices $p^*_t(\theta^g)$, $g = 1,...,N$. Under complete hedging against oil demand shocks households receive transfers: $a^g_{1t} = (p_t(\theta^g) - p^f_t)\theta^g k_t$, $-a^g_{2t} = (p^f_t - p_t(\theta^g))x_t$. The total payment to each country in state $g$ is $l_1 a^g_{1t} = (p_t(\theta^g) - p^f_t)\theta^g K_t = (p_t(\theta^g) - p^f_t)X_t = l_2 a^g_{2t}$. From proposition 6, the price of each contract is $\psi^{ig}_{t-1} = \tilde{\pi}^{ig}_t / R_t$, given that $\theta_{t-1} = \theta^i$, and the net trade of these contracts is

$$l_1 a_{1t} + l_2 a_{2t} = \qquad 2\sum_{g=1}^{N} \psi_t^{ig} \theta^g K_t (p_t^f - p_t^*(\theta^g)) = (2K_t / R_t) \sum_{g=1}^{N} \tilde{\pi}_t^{ig} \theta^g (p_t^f - p_t^*(\theta^g)) = 0$$

implying the forward oil price as

$$p_t^f = \frac{\sum_{g=1}^{N} \theta^g p_t^*(\theta^g) \tilde{\pi}_t^{ig}}{\sum_{g=1}^{N} \theta^g \tilde{\pi}_t^{ig}} = \frac{\tilde{E}_{t-1} \theta_t p_t^*(\theta_t)}{\tilde{E}_{t-1} \theta_t}.$$

This is the mean of state-contingent spot prices weighted with the risk-adjusted probabilities and oil input coefficients. The implicit forward oil price is determined in period $t-1$ and provides equalization of risk-adjusted expected factor returns similarly to (2.14): $\tilde{E}_{t-1} r(p_t^*(\theta_t)) / \delta + p_t^f \tilde{E}_{t-1} \theta_t = f'(k_t) - \tilde{E}_{t-1} D(\theta_t)$.

Complete risk sharing guarantees implementation of the global optimal path corresponding to some utility weight $\mu$, while optimal transfer is equal to state-contingent payment, $\tau_{jt}(\theta^g) = a_{jt}^g$. As in the deterministic case, an arbitrary utility weight can be matched by imposing the initial external debt $b_{j0}$ serviced through a flow of payments satisfying the intertemporal budget constraints for each economy.

### *3.4 Investments overseas and the global asset portfolio*

The assumption of complete markets can be essentially relaxed if we introduce cross-country investment in the production factors. Suppose that households in both countries make direct and indirect investments in the stocks of capital and oil and also trade in one-period risk-free bonds. Indirect investment is an amount of capital or oil stock acquired by a household in a perfect international market for production factors opened initially. For the sake of simplicity, the issues of corporate control are ignored and both kinds of investment into the household asset holdings are supposed to be perfect substitutes.

As in the basic model, firms and oil stocks are homogenous, and their numbers are $l_1$, $l_2$, respectively. The capital per firm $k_t$ and the oil stock per field $s_t$ are evolving according to the equations

$$k_t = (1-d)k_{t-1} + i_{1t-1}^{kd} + \lambda^{-1} i_{2t-1}^{kd},$$

$$s_t = s_{t-1} - x_{t-1} + (\lambda i_{1t-1}^{sd} + i_{2t-1}^{sd}) / \delta .$$

where $i_{jt-1}^{kd}$, $i_{jt-1}^{sd}$ are the amounts of direct investment in capital and oil stock, respectively, by a country $j = 1,2$ household. Labor is immobile and each manufacturing firm in country 1 employs one worker who is a resident of this country. The labor supply is inelastic and each employee is rewarded by the wage equal to $\omega_t = f(k_t) - f'(k_t)k_t$.

Let $k_{jt_t}$, $s_{jt_t}$ denote the volumes of capital and oil stock owned by a country $j$ household and $i^{ki}_{jt-1}$, $i^{si}_{jt-1}$ denote indirect investment by the household in acquisition of capital and oil stock, respectively. The household asset holdings evolve as

$$k_{jt} = (1-d)k_{jt-1} + i^{kd}_{jt-1} + i^{ki}_{jt-1},$$

$$s_{jt} = s_{jt-1} - x_{jt-1} + (i^{sd}_{jt-1} + i^{si}_{jt-1})/\delta .$$

where $x_{jt-1}$ are the volumes of oil extraction related to the oil stock holding by a country $j$ household satisfying: $x_{1t-1} = \lambda^{-1}x_{t-1}$, $x_{2t-1} = x_{t-1}$ (since $x_{t-1} = X_{t-1}/l_2$, $l_1 + l_2 = 1$). Production assets are purchased at market prices $\psi^k_{t-1}$ and $\psi^s_{t-1}$ – the Tobin's *Q* related to capital and oil stock, respectively.

Households choose the holdings of capital $k_{jt}$, oil stock $\hat{s}_{jt} = \delta s_{jt}$, bonds $b_{jt}$, direct and indirect investment $i^{kd}_{jt-1}$, $i^{sd}_{jt-1}$ and $i^{ki}_{jt-1}$, $i^{si}_{jt-1}$, by taking as given the factor prices $\psi^k_{t-1}$ and $\psi^s_{t-1}$, the wage $w_t$, the returns on investment $R^k_t$, $R^s_t$, and the risk-free bond return $R_t$. The household budget constraint in period $t-1$ is

$$\begin{aligned}&(\psi^k_{t-1}-1)i^{ki}_{jt-1} + (\psi^s_{t-1}-1)i^{si}_{jt-1} + k_{jt} + \hat{s}_{jt} + b_{jt} + c_{jt-1} = \\ &R^k_t k_{jt-1} + R^s_t \hat{s}_{jt-1} + R_t b_{jt-1} + \omega_{jt-1}\end{aligned} \qquad (3.13)$$

where $\omega_{jt} = \begin{cases}\omega_t, & j=1\\ 0, & j=2\end{cases}$ indicates that the wage is received only by country 1 households. The international financial markets are cleared if $\lambda i^{ki}_{1t} + i^{ki}_{2t} = \lambda i^{si}_{1t} + i^{si}_{2t} = \lambda b_{1t} + b_{2t} = 0$. In equilibrium the volumes of factors per firm/oil field are linked to the factor holdings by households as $k_t = k_{1t} + \lambda^{-1}k_{2t}$, $s_t = \lambda s_{1t} + s_{2t}$ (since $k_t = K_t/l_1$, $s_t = S_t/l_2$), and the global factor structure is equal to $\kappa_t = \dfrac{\lambda k_{1t} + k_{2t}}{\lambda s_{1t} + s_{2t}} = \dfrac{\lambda k_t}{s_t}$.

*Proposition 7. Under cross-country investment in factors and trade in bonds: i) the Tobin's Q are equal to one for both production factors,* $\psi^k_{t-1} = \psi^s_{t-1} = 1$; *ii) the risk-adjusted expected returns on portfolio and direct investment are equalized across factors and countries,* $j = 1,2$*:*

$$R_t = \tilde{E}_{jt-1}R^k_t = \tilde{E}_{jt-1}R^s_t; \qquad (3.14)$$

*iii) For a small oil extraction rate the global factor structure is near optimal:*

$$\kappa_t^2 \approx 2\delta \frac{f'(k_t) - d - \delta \widetilde{E}_{t-1}\theta_t}{\widetilde{E}_{t-1}\theta_t^2}. \qquad (3.15)$$

The Tobin's $Q$ are equal to one because direct and indirect investments are perfect substitutes for households. According to (3.14), the risk-adjusted expected returns on both factors coincide. The country subscript $j$ near the expectation operator in (3.14) means that the risk-adjusted probabilities, generally, differ between the countries since the interstate marginal rates of substitution differ. This is a potential source of inefficiency of investment decisions. Nevertheless, the second-order approximation obtained for a small extraction rate (3.15) shows that the two moments of risk-adjusted conditional distribution of $\theta_t$ coincide. From (3.14), the expected factor returns are the same across countries: $\widetilde{E}_{1t-1}R_t^k = \widetilde{E}_{2t-1}R_t^k$ and $\widetilde{E}_{1t-1}R_t^s = \widetilde{E}_{2t-1}R_t^s$ implying coincidence of the first- and second-order moments: $\widetilde{E}_{1t-1}\theta_t = \widetilde{E}_{2t-1}\theta_t \equiv \widetilde{E}_{t-1}\theta_t$ and $\widetilde{E}_{1t-1}\theta_t^2 = \widetilde{E}_{2t-1}\theta_t^2 \equiv \widetilde{E}_{t-1}\theta_t^2$. As a result, the global factor structure in (3.15) turns out to be the same as (3.7) providing near optimal production investment in the case of a small extraction rate.

In general, conditions of the risk-adjusted returns equalization (3.14) imply that the expected rates of return for both factors should be above the risk-free rate of return. The expected return is the sum of the risk-free return and the risk premium which is equal to the minus covariance of the marginal utility of consumption and the factor return divided by the mean marginal utility of consumption: $E_{t-1}R_t^l = R_t - Cov_{t-1}(u'_{jt}(c_{jt}), R_t^l) / E_{t-1}u'_{jt}(c_{jt})$ for $l = k, s$ and $j = 1,2$. From (3.14), this covariance should have the same sign for both countries and both factors. In trading equilibrium this sign is negative and the risk premium is positive for both factors. On the contrary, under conditions of financial autarky (3.1), (3.2), the returns on factors are negatively correlated implying that country 1 loses from a higher $\theta_t$ while country 2 gains. Under investment overseas the factor returns are positively correlated, as well as the marginal utilities of consumption in both countries. Households are enforced to compose investment risks in a similar manner indicating a high degree of international financial integration.

How does trade in factors bring about positive correlation in the returns to capital and oil stock, on the one hand, and in the marginal rates of interstate substitution across countries, on the other hand? To answer this question we reformulate the household problem as a dynamic portfolio selection problem in the following way. Let

$w_{jt} = R_t^k k_{jt} + R_t^s \hat{s}_{jt} + R_t b_{jt} + \omega_{jt}$ and $w'_{jt} = k_{jt} + \hat{s}_{jt} + b_{jt}$ denote the beginning- and end-of-period $t$ household disposable wealth of country $j$. Regarding direct and indirect investments as perfect substitutes, we can pool both kinds of investment by setting the Tobin's $Q$ to one for both factors. Then the budget constraint (3.13) is simplified as $c_{jt-1} = w_{jt-1} - w'_{jt}$, and the household dynamic portfolio problem is to maximize $v_j = E_0 \sum_{t=1}^{\infty} \beta^t u(w_{jt-1} - w'_{jt})$ by selecting a sequence of asset holdings $k_{jt}, \hat{s}_{jt}, b_{jt}$.

For this problem one can easily single out *a benchmark portfolio* of factor holdings which is the same for both economies and permits, in principle, a two-fund portfolio separation. Such a portfolio is built on the weights corresponding to the equilibrium factor structure of the global economy $\kappa_t$: $k_{jt}^G = \alpha(\kappa_t)G_{jt}$, $\hat{s}_{jt}^G = (1-\alpha(\kappa_t))G_{jt}$, where $\alpha(\kappa_t) = \kappa_t/(\delta + \kappa_t)$ is the weight of capital, $G_{jt}$ is the benchmark portfolio holding. This portfolio provides a complete mutual hedging of households against fluctuations in the marginal extraction cost. The return on the benchmark portfolio is $R_t^G(\theta_t) = 1 + [(f'(k_t) - D(\theta_t) - \phi^{-1}(\theta_t \kappa_t)/\kappa_t]\alpha(\kappa_t)$.[21] The term $\theta_t p_t(\theta_t)$ – the source of negative correlation in the factor returns $R_t^k(\theta_t)$ and $R_t^s(\theta_t)$ – is diversified away by taking the mutually offsetting positions in the capital and oil stock holdings.

Using the benchmark portfolio the household wealth is represented as $w'_{jt} = G_{jt} + g_{jt} + b_{jt}$, $w_{jt} = R_t^G G_{jt} + (R_t^k - R_t^s) g_{jt} + R_t b_{jt}$ where $g_{jt}$ is *a gambling portfolio* of country $j$ consisting of a long/short position in capital and a corresponding short/long position in oil stock. The return on this portfolio is equal to the gap of returns on capital and oil stock. In equilibrium $\lambda g_{1t} + g_{2t} = 0$, and holding this portfolio means essentially a kind of gambling between the countries, one of which gains at the expense of the other one. There exists a critical level of $\theta_t$ below which a country with long capital position gains from holding the gambling portfolio, while the other country losses and vice a versa.

Households in one of the countries have no choice but to reject from this gambling if households in the other country do the same. In a trading equilibrium with no gambling $g_{jt} = 0$ for $j = 1,2$ and all diversifiable oil price risks are eliminated. All non-diversifiable

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[21] Since $R_t^k(\theta_t) = 1 + f'(k_t) - D(\theta_t) - p_t(\theta_t)\theta_t$, $R_t^s(\theta_t) = 1 + r(p_t(\theta_t))/\delta$, the benchmark portfolio return is $R_t^G(\theta_t) = R_t^k(\theta_t)\alpha(\kappa_t) + R_t^s(\theta_t)(1-\alpha(\kappa_t)) = 1 + [(f'(k_t) - D(\theta_t) - \phi^{-1}(\theta_t\kappa_t)/\kappa_t]\alpha(\kappa_t)$. The marginal extraction cost term $p_t(\theta_t)\theta_t$ is eliminated from $R_t^G(\theta_t)$ since the static marginal oil rent is $r(p_t) = p_t\xi(p_t) - \phi^{-1}(\xi(p_t))$, and $\xi(p_t) = \theta_t\kappa_t$ under the oil market clearing.

risks are contained in the benchmark portfolio $G_{jt}$ and none of the countries benefits at the expense of the other one. Any cross-country variation in quantity of risks taken in asset trade is indicated by the relative weight of the benchmark portfolio (or bonds issued or purchased) in household wealth.

*Proposition 8. The benchmark portfolio return is represented as* $R_t^G(\theta_t) = R_t + \Delta R_t^G(\theta_t)$, *where* $\Delta R_t^G(\theta_t) = \alpha(\kappa_t)[\delta(\overline{\theta}_t - \theta_t) + (\phi^{-1}(\overline{\theta}_t\kappa_t) - \phi^{-1}(\theta_t\kappa_t))/\kappa_t]$ *is the excess return, and* $\overline{\theta}_t$ *is a threshold oil input coefficient such that* $\Delta R_t^G(\overline{\theta}_t) = 0$.

The excess return on the benchmark portfolio $\Delta R_t^G(\theta_t)$ is composed of gains or losses from movements of the oil input coefficient $\theta_t$ relative to the threshold level of this coefficient $\overline{\theta}_t$ ensuring certainty equivalence: $R_t^G(\overline{\theta}_t) = R_t = 1 + \alpha(\kappa_t)[f'(k_t) - D(\overline{\theta}_t) - \phi^{-1}(\overline{\theta}_t\kappa_t)/\kappa_t]$. The excess return is obtained from changes in the marginal cost of oil stock maintenance, $\delta(\overline{\theta}_t - \theta_t)$, and in the cost of oil extraction per barrel of oil in the ground, $\phi^{-1}(\overline{\theta}_t\kappa_t) - \phi^{-1}(\theta_t\kappa_t)$, relative to the threshold level.

Such a representation of the benchmark portfolio return is appealing for a special case of Markov chain with a quasi-diagonal transition matrix: $\pi^{ig} > 0$ for $g = i-1, i, i+1$ and $\pi^{ig} = 0$ otherwise, provided that $1 < i < N$, $N \geq 3$. For such a chain the oil demand parameter $\theta_t$ can move from one period to the next only to the neighbor states or remain in the same state. If the variation of oil input coefficient $\theta^{i+1} - \theta^i$ is quite small for any state, then the excess return on the benchmark portfolio is:[22]

$$\Delta R_t^G(\theta_t) \approx \alpha(\kappa_t) P_t^f(\overline{\theta}_t)\Delta\overline{\theta}_t,$$

where $\Delta\overline{\theta}_t = \overline{\theta}_t - \theta_t$, $P_t^f(\overline{\theta}_t) = \delta + p_t(\overline{\theta}_t)$, and $\overline{\theta}_t \in (\theta_{i-1}, \theta_{i+1})$. Households in both countries obtain a positive (negative) excess return in period $t$, if the oil input coefficient turns out to be below (above) the threshold level, $\theta_t < (>) \overline{\theta}_t$. The benchmark portfolio rewards or punishes investors for oil demand changes relative to this level measured as $\alpha(\kappa_t)\Delta\overline{\theta}_t$ and priced through the forward oil price $P_t^f(\overline{\theta}_t)$. The latter is based on the threshold oil input coefficient $\overline{\theta}_t$ and the equilibrium factor structure $\kappa_t$ both known in period $t-1$. The higher

[22] We have: $\phi^{-1}(\overline{\theta}_t\kappa_t) - \phi^{-1}(\theta_t\kappa_t) \approx z'(\overline{\theta}_t\kappa_t)\Delta\overline{\theta}_t\kappa_t = \varphi(\overline{\theta}_t\kappa_t)\Delta\overline{\theta}_t\kappa_t = p_t(\overline{\theta}_t)\Delta\overline{\theta}_t\kappa_t$. Then $\Delta R_t^G(\theta_t) = \alpha(\kappa_t)[\delta(\overline{\theta}_t - \theta_t) + (\phi^{-1}(\overline{\theta}_t\kappa_t) - \phi^{-1}(\theta_t\kappa_t))/\kappa_t] \approx \alpha(\kappa_t)(\delta + p_t(\overline{\theta}_t))\Delta\overline{\theta}_t = \alpha(\kappa_t)P_t(\overline{\theta}_t)\Delta\overline{\theta}_t$.

is this price, the larger is the mean excess return $E_{t-1}\Delta R_t^G(\theta_t)$ providing thereby stronger incentives to invest in the production factors.

As a result, the benchmark asset portfolio rewards households in both countries for energy-economizing shifts of production technology. Moreover, the return on this portfolio encourages risky investment in the case if the oil demand in current period is high relative to the oil stock. The structure of risks contained in the benchmark portfolio is defined by the global factor structure and identical for both countries. Holding of this portfolio under asset trade ensures that the marginal rates of interstate substitution, as well as the asset returns, are positively correlated across countries.

## Concluding remarks

The global economy is energy-dependent because energy consumption is comprehensive, capital and energy are to a large extent complementary production factors, and international trade in the energy-careers is highly specialized. The model of this paper reflects these features, though in a stylized form, and emphasizes a dynamic interdependence between capital accumulation and oil stock extension. As has been shown, financial openness under energy-dependence is necessary for optimal investment in the production factors separated by the national border. This is a hardly unexpected result since the intertemporal and interstate consumption smoothing rests on the equalization of expected factor returns. What, in our view, imparts some novelty to this condition of optimality is that the oil price plays in our model the key role underlying the equalization of returns and the determination of optimal factor structure in the global economy.

The model extension to the case of uncertainty in the oil demand and cross-country investment concerned the effects of partial risk sharing between the countries. Free trade in production factors eliminates negatively correlated country-specific risks and promotes households to build portfolios composed of only non-diversifiable global risks which are the same for both economies. Though the marginal rates of interstate substitution are, in general, not equalized across the countries, the configuration of risks in the households' portfolios of wealth is closely related. The benchmark global asset portfolio corresponds to the global factor structure and captures the essence of international financial integration. Households take perfectly correlated risks related to the costs of oil stock maintenance and oil extraction which are combined in the benchmark portfolio return. Individuals in different countries obtain the same investment opportunities and their citizenship is irrelevant for their choice of risky asset holdings. Under the no-gambling trading equilibrium (the case $g_{jt} = 0$) the composition of risky asset and debt in household portfolio varies across countries because of the variation in risk attitude which depends on the wealth per capita.

The initial wealth distribution is determined in this model as trade in assets is opened, through an initial swap of production factors. As a further development of the model one can suggest a long-term debt contract underlying implementation of this swap. Such a contract involves wealth management on the international basis aimed at the interstate equalization of the relative marginal value of wealth. This is possible under the no-gambling trading equilibrium since the household portfolio in both countries is composed of the same benchmark risky asset and the risk-free debt. An important issue is the heterogeneity of wealth structure of these countries caused by the absence of labor income in country 2. This heterogeneity can be eliminated through the initial debt issue by country 1 generating a flow of subsequent debt payments to country 2 perfectly correlated with labor income. This kind of contract model may be useful for reflection on the role of international financial institutions in internalizing the effects of global factor accumulation on external debts.

Though the general implication of this paper model is basically normative, it may be also used as a tool for a positive theory of international finance. The empirical evidence suggests that the extent of international portfolio diversification among industrial economies is typically low. In the case of oil-producing and oil-consuming economies this evidence is explained, on the surface, by the official restrictions on cross-border investment that were mentioned in the introduction. But opening of investment flows may, hypothetically, not lead to a close financial integration of these economies. The existence of a gambling equilibrium in the model with asset trade (the case $g_{jt} \neq 0$) could provide a prediction that households in these economies would not necessarily diversify away the negatively correlated risks if the existing barriers to invest were completely removed. If this inference of the model turns out to be true, it could be attributed to the model fundamentals like the risk preferences of households.

## Appendix

*Proposition 1*. The Euler equation for country 1 is: $u'(c_{1t-1})\dfrac{\partial c_{1t-1}}{\partial k_t}+\beta u'(c_{1t})\dfrac{\partial c_{1t}}{\partial k_t}=0$. We have from (1.2), (1.3): $\partial c_{1t-1}/\partial k_t=-1$, and, from (1.16), $\partial c_{1t}/\partial k_t=1+f'(k_t)-D-\theta p_t$ implying $u'(c_{1t-1})=\beta(1+f'(k_t)-D-\theta p_t)u'(c_{1t})$. Similarly, this equation for country 2 is: $u'(c_{2t-1})\dfrac{\partial c_{2t-1}}{\partial \widehat{s}_t}+\beta u'(c_{2t})\dfrac{\partial c_{2t}}{\partial \widehat{s}_t}=0$. From (1.2), (1.3) we obtain: $\partial c_{2t-1}/\partial \widehat{s}_t=-1$, and from (1.17), $\partial c_{2t}/\partial \widehat{s}_t=1+r(p_t)/\delta$, implying $u'(c_{2t-1})=\beta(1+r(p_t)/\delta)u'(c_{2t})$.

*The global planner problem (2.1)-(2.6):* The Lagrangian for this problem is $L^{\mu}=\sum_{t=0}^{\infty}\beta\big(\mu l_1 u(c_{1t})+l_2 u(c_{2t})+\eta_t(\theta K_t-X_t)\big)$, where $\eta_t$ is a dual variable related to (2.3). We have, from (2.2), (2.4)-(2.6) that $c_{1t}=(F_t-(P_t\theta K_t-(K_{t+1}-K_t+dK_t)-T_t)/l_1$, $c_{2t}=(P_tX_t-Z_t-\ \delta(S_{t+1}-S_t+X_t)+T_t)/l_2$. The first-order condition for the oil price $P_t$ is $\mu\theta K_t u'(c_{1t})=X_t u'(c_{2t})$ or $\mu u'(c_{1t})=u'(c_{2t})$ for (2.3) held as equality. The same first-order condition is obtained for $T_t$.

The first-order condition for oil extraction is $P_t=\partial z/\partial x_t+\delta+\eta_t'\ =p_t+\delta+\eta_t'$, where $\eta_t'=\eta_t/u'(c_{2t})$ is the dual variable in the final good units. The constraint (2.3) holds identically as equality for oil rent-maximizing oil supply $X_t=S_t\xi(p_t)$ implying that $p_t=\varphi(\theta_t K_t/S_t)$ and $\eta_t'=0$. The optimal oil price is $P_t=p_t+\delta$.

*Proposition 2.* From the capital accumulation equation (2.8), $\partial C_{1t}/\partial K_t=$ $1+\partial F/\partial K_t-D-\theta p_t-\theta K_t\partial p_t/\partial K_t=R_t^k-\theta K_t\theta\varphi'(\theta\kappa_t)/S_t=R_t^k-\theta^2\varphi'(\theta\kappa_t)\kappa_t$, because $\partial F/\partial K_t=f'(k_t)$ and $p_t=\varphi(\theta\kappa_t)$. From the oil stock accumulation equation (2.9), $\partial C_{2t}/\partial K_t=(\widehat{S}_t/\delta)dr(p_t)/dp_t\cdot\partial p_t/\partial K_t=S_t\xi(p_t)\theta\varphi'(\theta\kappa_t)/S_t=\theta^2\varphi'(\theta\kappa_t)\kappa_t$ because $r'(p_t)=\xi(p_t)$, and $S_t\xi(p_t)=\theta K_t$ from the oil market-clearing condition (1.15).

Similarly, from (2.9), $\partial C_{2t}/\partial\widehat{S}_t=1+r(p_t)/\delta\ +(\widehat{S}_t/\delta)dr(p_t)/dp_t\cdot\partial p_t/\partial\widehat{S}_t=$ $R_t^s-S_t\xi(p_t)\theta\varphi'(\theta\kappa_t)K_t/S_t^2\delta=R_t^s-\theta\xi(p_t)\varphi'(\theta\kappa_t)\kappa_t/\delta$, and from (2.8),

$\partial C_{1t} / \partial \widehat{S}_t = -\theta K_t \partial p_t / \partial \widehat{S}_t = \theta K_t \theta \varphi'(\theta \kappa_t) K_t / S_t^2 \delta = \theta \xi(p_t) \varphi'(\theta \kappa_t) \kappa_t / \delta$, because $\theta K_t / S_t = \xi(p_t)$.

*Proposition 3.* The Euler equation for investment in capital is

$$\mu l_1 \left[ u'(c_{1t-1}) \frac{\partial c_{1t-1}}{\partial K_t} + \beta u'(c_{1t}) \frac{\partial c_{1t}}{\partial K_t} \right] + l_2 \left[ u'(c_{2t-1}) \frac{\partial c_{2t-1}}{\partial K_t} + \beta u'(c_{2t}) \frac{\partial c_{2t}}{\partial K_t} \right] = 0 \quad \text{(A2.1)}$$

The capital accumulation equation (2.8) implies that $\partial c_{1t-1} / \partial K_t = l_1^{-1} \partial C_{1t-1} / \partial K_t = -l_1^{-1}$. From (2.10), $\partial c_{1t} / \partial K_t = l_1^{-1} \partial C_{1t} / \partial K_t = (R_t^k - \Delta R_t^k) l_1^{-1}$, $\Delta R_t^k = \theta^2 \varphi'(\theta \kappa_t) \kappa_t$. Similarly, from (2.9) and (2.10) we have that $\partial c_{2t-1} / \partial K_t = 0$, $\partial c_{2t} / \partial K_t = l_2^{-1} \partial C_{2t} / \partial K_t = \Delta R_t^k l_2^{-1}$. Inserting the marginal consumption rates into (A2.1) yields:

$$\mu \left[ - u'(c_{1t-1}) + \beta u'(c_{1t})(R_t^k - \Delta R_t^k) \right] + \beta u'(c_{2t}) \Delta R_t^k = 0,$$

or $u'(c_{1t-1}) = \beta R_t^k u'(c_{1t}) - \beta \Delta R_t^k p_t (u'(c_{1t}) - \mu^{-1} u'(c_{2t}))$. Due to (2.7), the second term on the right-hand side of this equation is zero implying $u'(c_{1t-1}) = \beta R_t^k u'(c_{1t})$.

The Euler equation for investment in oil stock is

$$\mu l_1 \left[ u'(c_{1t-1}) \frac{\partial c_{1t-1}}{\partial \widehat{S}_t} + \beta u'(c_{1t}) \frac{\partial c_{1t}}{\partial \widehat{S}_t} \right] + l_2 \left[ u'(c_{2t-1}) \frac{\partial c_{2t-1}}{\partial \widehat{S}_t} + \beta u'(c_{2t}) \frac{\partial c_{2t}}{\partial \widehat{S}_t} \right] = 0. \quad \text{(A2.2)}$$

Equations (2.9) and (2.11) imply that $\partial c_{1t-1} / \partial \widehat{S}_t = 0$, $\partial c_{1t} / \partial \widehat{S}_t = l_1^{-1} \partial C_{1t} / \partial \widehat{S}_t = \Delta R_t^s l_1^{-1}$, where $\Delta R_t^s = \theta \xi(p_t) \varphi'(\theta \kappa_t) \kappa_t / \delta$, and $\partial c_{2t-1} / \partial \widehat{S}_t = l_2^{-1} \partial C_{2t-1} / \partial \widehat{S}_t = -l_2^{-1}$, $\partial c_{2t} / \partial \widehat{S}_t = l_2^{-1} \partial C_{2t} / \partial \widehat{S}_t = (R_t^s - \Delta R_t^s) l_2^{-1}$. Inserting these derivatives into (A2.2) yields:

$$\mu \beta u'(c_{1t}) \Delta R_t^s - u'(c_{2t}) + \beta u'(c_{2t})(R_t^s - \Delta R_t^s) = 0$$

or, rearranging terms, $u'(c_{2t-1}) = \beta R_t^s u'(c_{2t}) + \beta \Delta R_t^s \left( \mu u'(c_{1t}) - u'(c_{2t}) \right)$. The second term on the right-hand side of this equation is zero and, as a result, (A2.2) is equivalent to $u'(c_{2t-1}) = \beta R_t^s u'(c_{2t})$.

*Proposition 4. (i)* The stochastic Euler equation for capital is

$$\mu l_1 \left[ u'(c_{1t-1}) \frac{\partial c_{1t-1}}{\partial K_t} + \beta E_{t-1} u'(c_{1t}) \frac{\partial c_{1t}}{\partial K_t} \right] + l_2 \left[ u'(c_{2t-1}) \frac{\partial c_{2t-1}}{\partial K_t} + \beta E_{t-1} u'(c_{2t}) \frac{\partial c_{2t}}{\partial K_t} \right] = 0.$$

The marginal consumption rates are defined for any state in the same way as in proposition 2 for the deterministic model. Taking into account the condition of weighted marginal utilities equalization in any state (2.7) yields the Euler equation (2.12) in the same way is in the proof of proposition 3. The stochastic Euler equation for oil stock (2.13) is derived similarly.

*ii)* Condition (3.3) implies that for any $g = 1,...,N$:

$$\frac{u'(c_{1t}(\theta^g))}{\sum_{d=1}^{N}\pi^{id}u'(c_{1t}(\theta^d))} = \frac{u'(c_{2t}(\theta^g))}{\sum_{d=1}^{N}\pi^{id}u'(c_{2t}(\theta^d))},$$

$\theta_{t-1} = \theta^i$, which can be written as $u'(c_{1t}(\theta^g))/E_{t-1}u'(c_{1t}) = u'(c_{2t}(\theta^g))/E_{t-1}u'(c_{2t})$ and means equalization of risk-adjusted transitional probabilities: $\tilde{\pi}_{1t}^{ig} = \tilde{\pi}_{2t}^{ig}$ for $i, g = 1,...,N$.

*iii)* The Euler equations (3.4) can be combined as

$$\frac{E_{t-1}R_t^k u'(c_{1t})}{u'(c_{1t-1})} = \frac{E_{t-1}R_t^s u'(c_{2t})}{u'(c_{2t-1})}. \qquad \text{(A3.1)}$$

From (3.3), the ratios of marginal utilities equalize between countries for $i, g = 1,...,N$:

$$\frac{u'(c_{1t}(\theta^g))}{u'(c_{1t-1}(\theta^i))} = \frac{u'(c_{2t}(\theta^g))}{u'(c_{2t-1}(\theta^i))}.$$

Substituting for these ratios in (A3.1) for each state $g$ yields

$$E_{t-1}R_t^k u'(c_{jt}) = E_{t-1}R_t^s u'(c_{jt}),$$

for $j = 1,2$. Dividing this equation by $E_{t-1}u'(c_{jt})$ yields $\tilde{E}_{t-1}^j R_t^k = \tilde{E}_{t-1}^j R_t^s$ or, since the risk-adjusted conditional expectations coincide for $j = 1,2$, $\tilde{E}_{t-1}R_t^k = \tilde{E}_{t-1}R_t^s$.

*Proposition 5.* The Taylor series expansion of the second order for the left-hand side of (3.6) yields: $\tilde{E}_{t-1}[r(\varphi(\theta_t\kappa_t))/\delta + \theta_t\varphi(\theta_t\kappa_t) + \theta_t\delta] \approx \tilde{E}_{t-1}[r(\varphi(0))/\delta + \theta_t\varphi(0) +$

$$r'(0)\varphi'(0)\theta_t\kappa_t/\delta + \theta_t\varphi'(0)\theta_t\kappa_t + \frac{d^2r(\varphi(0))}{d(\theta_t\kappa_t)^2}(\theta_t\kappa_t)^2/2\delta + \theta_t\varphi''(0)(\theta_t\kappa_t)^2/2\delta + \theta_t\delta] =$$

$$\tilde{E}_{t-1}[\varphi'(0)\theta_t^2\kappa_t + \frac{d^2r(\varphi(0))}{d(\theta_t\kappa_t)^2}(\theta_t\kappa_t)^2/2\delta + \theta_t\varphi''(0)(\theta_t\kappa_t)^2/2\delta + \theta_t\delta],$$ since

$r(0) = \varphi(0) = r'(0) = 0$. We have: $\varphi'(0) = 1/\xi'(0) = 0$, $\varphi''(0) = -\xi''(0)/\xi'(0)^3 = 0$, $d^2r(\varphi(0))/d(\theta_t\kappa_t)^2 = r''(0)\varphi'(0) + r'(0)\varphi''(0) = r''(0)\varphi'(0) = \xi'(0)/\xi'(0) = 1$. As a result, the right-hand side of (3.6) is approximated as $\tilde{E}_{t-1}[(\theta_t\kappa_t)^2/2\delta + \theta_t\delta] = (\kappa_t^2/2\delta)\tilde{E}_{t-1}\theta_t^2 + \delta\tilde{E}_{t-1}\theta_t$, and (3.6) is represented as

$(\kappa_t^2/2\delta)\tilde{E}_{t-1}\theta_t^2 + \delta\tilde{E}_{t-1}\theta_t = f'(k_t) - d$, implying $\kappa_t^2 = 2\delta\dfrac{f'(k_t) - d - \delta\tilde{E}_{t-1}\theta_t}{\tilde{E}_{t-1}\theta_t^2}$.

*Proposition 6*. The first-order condition for the state-$g$ contingent claim is

$$\psi_{t-1}^{ig} u'(c_{jt-1}(\theta^i)) = \beta\pi^{ig} u'(c_{jt}(\theta^g)), \qquad \text{(A3.2)}$$

for $j = 1,2$, $\theta_{t-1} = \theta^i$, $\theta_t = \theta^g$, and $i, g = 1,...,N$. Summing both sides of (A3.2) over states $g$ implies: $u'(c_{jt-1}) = \beta R_t E_{t-1} u'(c_{jt})$, where $R_t = 1/\sum_{g=1}^{N} \psi_{t-1}^{ig}$. Inserting this into (A3.2) yields: $\psi_{t-1}^{ig} = R_t^{-1}\pi^{ig} u'(c_{jt}(\theta^g))/E_{t-1}u'(c_{jt})$ or $\tilde{\pi}_{jt}^{ig} = \psi_{t-1}^{ig} R_t$, which means equalization of risk-adjusted transition probabilities between the countries: $\tilde{\pi}_{1t}^{ig} = \tilde{\pi}_{2t}^{ig} \equiv \tilde{\pi}_t^{ig}$. Equalization of risk-adjusted expected returns (3.12) is shown in the same way as for proposition 5.

*Proposition 7*. The Euler equations for direct and indirect investment in production factors are $u'(c_{jt-1}) = \beta E_{t-1} R_t^l u'(c_{jt})$, $\psi_{t-1}^l u'(c_{jt-1}) = \beta E_{t-1} R_t^l u'(c_{jt})$, respectively, $l = k, s$, $j = 1,2$. These equations are compatible only if $\psi_{t-1}^l = 1$. The equations for bonds are: $u'(c_{jt-1}) = \beta R_t E_{t-1} u'(c_{jt})$. Combining all these equations yields: $R_t = E_{t-1} R_t^l u'(c_{jt})/E_{t-1}u'(c_{jt})$ or $R_t = \tilde{E}_{jt-1} R_t^k = \tilde{E}_{jt-1} R_t^s$.

The latter equation is similar to (3.6), and yields the similar second-order approximation: $(\kappa_t^2/2\delta)\tilde{E}_{jt-1}\theta_t^2 + \delta\tilde{E}_{jt-1}\theta_t \approx f'(k_t) - d$. Since the expected risk-adjusted returns are the same across countries, we have $\tilde{E}_{1t-1} R_t^k = \tilde{E}_{2t-1} R_t^k$, $\tilde{E}_{1t-1} R_t^s = \tilde{E}_{2t-1} R_t^s$ implying $\tilde{E}_{1t-1}\theta_t = \tilde{E}_{2t-1}\theta_t \equiv \tilde{E}_{t-1}\theta_t$ and $\tilde{E}_{1t-1}\theta_t^2 = \tilde{E}_{2t-1}\theta_t^2 \equiv \tilde{E}_{t-1}\theta_t^2$, respectively, for our approximation. The first- and second-order risk-adjusted moments of $\theta_t$ coincide, and we obtain (3.15).

*Proposition 8*: proof is straightforward.

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