THE EXTENT OF THE MARKET AS A FACTOR OF GROWTH

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Abstract. According to cross-country evidence, the effect of openness on growth is positive but diminishing with country size, the average firm size is positively correlated with the market size, and the observed relationship of firm size and country size is U-shaped (for the EU countries). The paper tries to link these pieces of evidence together and suggests that the positive effect of openness on growth is diminishing with country size because of the positive effects of market extension on average firm size. Reduction of trade barriers leads to extension of international markets strengthening competitive pressure on domestic firms. Exits and mergers caused by international competition reduce the number of surviving firms and increase their size. As a result, the survivals have stronger incentives to generate technological shifts basing on higher profits that enhance financial opportunities for investment in new technology. This competitive pressure effect of international trade on the firms' size is more pronounced for a smaller economy. Firms therein have more powerful incentives to increase dynamic efficiency resulting in a stronger growth effect of openness.

1. Introduction

According to empirical evidence, the country size is more important for growth under obstacles to foreign trade, while openness is more beneficial for small economies. As has been established by Alberto Ades & Edward Glaeser 1999, Alberto Alesina et al. 2000, 2004, Francisco Alcala & Antonio Ciccone 2003, a reduction of trade barriers has a stronger effect on a small economy growth. These studies build on cross-country growth regressions including openness and country size as explanatory variables and demonstrating that the positive effect of openness on growth is diminishing with country size.

Cross-country evidence also indicates a positive link between openness and average firm size in an economy. For instance, as shown by Krishna Kumar et al. (1999), firm size is significantly correlated with market size. Removing political and other barriers to trade tends to increase average firm size as, for example, reported by Reinhilde Veugelers (2002, p. 123) and Fabienne Ilzkovitz et al. (2007, p. 44) for the case of European integration (figure 1). This case provides a nice historical example of permanent reduction of trade barriers within a community of developed and closely linked economies. The average firm size for a sample of 225 largest EU firms nearly doubled, from 2.9 to 5.7 billion euro, over the period 1987-2000.

On the other hand, average firm size ambiguously relates to home country size, as seen from figure 2, where average number of employees per manufacturing firm and total population size are plotted for a sample of 20 European economies in 2000 and 2004. The relationship is clearly U-shaped: the firm size is bigger for large economies as well as for small ones. The latter include countries like Belgium, Finland, Ireland, Norway and Denmark with dominating position of globally-oriented large firms[1].

Figures 1 and 2 here

[1] Leslie Hannah (1996) identifies transnational corporations from the world top list of 100 with headquarters in Belgium, Canada, Netherlands, New Zealand, and Sweden as “significantly global” rather than “nationally focused”.

This paper tries to link these three pieces of evidence and suggests that the positive effect of openness on growth is mediated by the positive effect of market extension on average firm size. The basic idea is that reduction of trade barriers leads to extension of international markets strengthening competitive pressure on domestic firms. Exits and mergers caused by international competition reduce the number of surviving firms and increase their size. As a result, the survivals have stronger incentives to generate technological shifts basing on higher profits that enhance financial opportunities for investment in new technology. The competitive pressure effect of international trade on the firms' size is more pronounced for smaller economies. Firms therein have more powerful incentives to raise dynamic efficiency, implying that the growth effect of openness is stronger for small economies and diminishing with country size, as indicated by the abovementioned cross-country growth regressions.

In addressing this issue we suggest a theoretical model of multilateral trade and technology growth based on investment decisions by firms that produce final goods under increasing returns to scale. Firms compete for market niches in the global market and invest in production technology resulting in growth of labor productivity or quality of goods. National knowledge spillovers are raising effectiveness of these investments and positively conditioned upon the size of domestic economy. The model is essentially a dynamic extension of Paul Krugman's (1979, 1980) basic framework for intraindustry trade with monopolistic competition of firms. It also relates to the well-established endogenous growth models with increasing returns to scale, horizontal differentiation of goods and bilateral trade by Luis Rivera-Batiz and Paul Romer (1991), Gene Grossman and Elhanan Helpman (1991), Robert Feenstra (1996). Points of departure from these and other models of growth and trade are specified in the next section.

The key message of our model is that the extent of the market, being properly defined, can be viewed as a factor facilitating technological competition and growth for all trading economies. The equilibrium growth rate of technology is shown to be determined as the product of *country-*

size and *market-size factors*. The former captures the effect of national knowledge spillovers, while the latter indicates a degree of competitive pressure on a domestic firm as the inverse of the global market share of the domestic economy. The market-size factor of growth is thus shown to be more important for a relatively small or less advanced country with a lower share in the global market. Under barriers to trade, this factor is adjusted for a measure of the firm openness to global trade, *the market extent*, derived for the multi-country world with fragmented national markets. This measure of openness indicates a degree of correlation of domestic firms' shares in the national markets – the local market shares varying because of trading costs. Reduction of trade barriers leads to an increase of the market extent for the firm and the market-size factor of growth for the national economy. In the long term this factor depends eventually on the model fundamentals: the world structure of trading costs and the size distribution of nations.

Because of the trade barriers and country size differentiation, aggregate price indices vary across countries. The equilibrium real exchange rate (the relative consumer price index) is defined for the stationary balanced growth path, and the closed-form solution is examined for the case of two asymmetric countries. This rate depends on the exogenous model parameters of relative country size and openness of trade flows (the inverse of trade costs) and, in turn, defines the steady-state market size factor of growth. The latter is shown to be closely related to the abovementioned empirical findings by Alesina et al. 2000, 2004, Alcala & Ciccone 2003. As in those regressions, the long-term growth rate is positively linked to country size and openness, and negatively – to their product. Consistently with empirical evidence, the positive effect of openness on growth is diminishing with the relative country size and, similarly, the country-size effect is weakening with openness.

The next section contains brief review of related literature. Section 3 presents the basic model, and section 4 provides results for the common market case. Section 5 extends the model to include trading costs and derives the main results of the paper, and section 6 examines the stationary growth path and the issue of real exchange rates determination. Sections 7 and 8 are devoted to informal discussion of results and concluding remarks. Figures and proofs are collected in Appendices.

2. Links to literature

The extent-of-the-market theory of growth is rooted in Adam Smith's assertion (1776) that the division of labor, specialization and economic progress are constrained by the barriers to trade. Often-mentioned chapter 3 of his study entitled "That the division of labor is limited by the extent of the market" is devoted largely to the role of trading costs reduction in extension of markets and economic development: "As by means of water carriage a more extensive market is opened to every sort of industry than what land-carriage can afford it, so it is upon the sea-cost, and along the banks of navigable rivers, that industry of every kind begins to subdivide and improve itself." Smith describes in this chapter historical examples of ancient Mediterranean and Asian civilizations that flourished due to the advantages of water transportation.[2]

The idea that market extension drives economic progress became attractive, especially since Allyn Young (1928) made "Adam Smith's theorem" the central theme of his analysis of increasing returns to scale.[3] Subsequent research focused on the nature of increasing returns under imperfect competition and resulted in the intellectual breakthroughs of the new theories of trade and growth of the 1980-s.[4] The Pin Factory and the Invisible Hand parables of "The Wealth of Nation" were ultimately reconciled (Warsh 2006), but the key Smith's proposition stated as the title of chapter 3 has not received conclusive support from the formal models with increasing

[2] George Stigler (1951, p. 192) remarked that "after all, reductions of transportation costs are a major way of increasing the extent of the market".

[3] Young shared the Smith's view on trade and growth: "It is dangerous to assign any single factor the leading role in that continuing economic revolution which has taken the modern world so far away from the world of a few hundred years ago. But is there any other factor which has a better claim to that role than the persisting search for the markets? No other hypothesis so well unites economic history and economic theory" (Young, 1928, p. 536).

[4] The seeds of this breakthrough were sown by Alfred Marshall (1890) who suggested the ideas of external and internal economies and the falling demand curve for the individual firm. The reviews of how these and related concepts evolved has been presented by Sylvia Peon (2003) and David Warsh (2006).

returns. National or international externalities arising from investment in human capital or R&D have been shown to influence ambiguously growth in trading economies through effects of specialization, learning-by-doing or cross-country spillovers.[5]

This paper, on the contrary, makes focus on the original Smith's conjecture that trading costs reduction implies market extension that favors economic progress. As our model infers, market size, average firm size and technology growth of any country are enhancing with reduction of trade barriers related to this country. Noteworthy, the worldwide trading costs structure has been essentially ignored in the well-established models of trade and growth. The effects of trade on growth are usually demonstrated through extensions of closed-economy models to the world economy with two countries and no trading costs. Growth rates are compared for two extreme regimes – autarky and complete openness. Such a simple dichotomy of isolated and integrated economies is inevitable for growth models with no trading costs, but is, generally, improper for treating the extent of the global market as a factor of growth.

Another essential difference is that growth equation in our model is implied from the free-entry condition eliminating net profits of firms, instead of the labor market-clearing condition. The latter is the case in the standard endogenous growth models with expanding intermediate varieties proposed originally by Paul Romer (1990), and Gene Grossman and Elhanan Helpman (1991). Newly established intermediate firms are the synonyms of blueprints produced by domestic R&D sector, and the cost of new entry is the equivalent of the patent price rewarding research activity. Correspondingly, growth in the standard models is determined by the amount of national labor force allocated to this sector. In our model the number of production firms is determined through entry-exit-merger decisions, and technology growth is defined by the allocation of labor force within a firm.

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[5] In the endogenous growth models with trade pioneered by Rivera-Batiz & Romer (1991) and Helpman & Grossman (1991), integration may increase productivity growth in all countries through positive external effects of international knowledge spillovers. In the models with learning-by-doing (Krugman 1987, Young 1991, Stokey 1991) trade may slow growth in these economies because of unfavorable specialization and shifts in industrial structure.

This implies dramatic distinction in the predicted effects of international trade on growth. The inference of the Rivera-Batiz & Romer model (1991) with two symmetric countries and knowledge-based research is that trade in goods does not affect growth because it does not change the allocation of labor between manufacturing and R&D. This "knife-edge" result holds only for the case of identical initial stocks of knowledge, as was shown by Michael Devereux & Beverly Lapham (1994). Otherwise, opening of international markets has negative impact on growth of a less advanced economy. Grossman and Helpman (1991, chapter 8) demonstrated a hysterisis effect of trade implying prescription "once behind, always behind" for a two-country model of growth.[6] Only if the lagged country is sufficiently large in relative size, it can catch up the leader. A version of this model was examined for the case of country-size asymmetry by Robert Feenstra (1996). He shows that opening of trade is harmful for growth of a smaller economy[7], where the relative wage declines inducing reduction in the value of R&D and reallocation of labor from research activity. These results are at odds with the cross-country evidence on the market extent as a factor of growth referred to in the introduction. By contrast, in our model the extension of markets causes the firm size to increase in any economy and provides incentives to devote more labor to technology growth on the firm level. This effect is more pronounced for a smaller (or less advanced) economy than for a larger one.

The standard models, thus, predict divergence of growth rates and can feature stabilizing dynamics only due to the crucial assumption of international knowledge spillovers. For instance, unlike trade in goods, international flow of knowledge in the Rivera-Batiz & Romer model (1991) causes positive effects on growth for both economies. But as Feenstra (1996, p. 229) points it out, "If instead we drop this assumption, and consider the case where no diffusion of knowledge

[6] The divergence effects of opening markets are more pronounced, if the issues of dynamic comparative advantage and unfavorable industry specialization are taken into consideration, as in Grossman and Helpman (1990, 1991). Similarly, trade can reinforce skill bias in technical change which is unfavorable for a less advanced economy and is induced by the relative price effect rather than by the market-size effect, according to the theory of directed technical change of Daron Acemoglu (2002).

[7] For the usual case when foreign and domestic consumer goods are substitutes.

occurs, then we find that trade in goods can lead to a divergence of growth rates". Feenstra offers arguments (op. cit., p. 231) justifying the assumption of his basic model that R&D knowledge does not cross borders (technological differences among countries, substantial lags in the flow of technical knowledge across borders, high social returns to R&D that are found nationally). In our model we ignore the effects of international knowledge transfers as well, and capture in the extreme form, similarly to Feenstra (1996), the limited cross-border diffusion of technology.

Our emphasis on investments in technology by firms as a source of growth is justified by theory and empirical evidence of modern industrial organization. Market structure in many industries is dependent upon endogenous fixed costs like investments by firms in R&D or advertising. The effects of these costs on the concentration-market size relation have been examined by Shaked and Sutton (1987) and Sutton (1991). Lyons et al. (2001) have found strong support for importance of endogenous fixed costs as market structure determinant in the case of European integration. They remark that "Such investments tend to raise consumer perception of a product quality, and so can be powerful competitive weapons in the battle for market share" (Lyons et al. 2001, p. 3). As market extends, firms get incentives to expand their shares in the industry by making investments that increase endogenous fixed costs. These investments are therefore responsive to the size of the market, and industries that compete using such costs exhibit significantly higher the limiting lower bound on concentration than do industries that compete mainly on price (Lyons et al. 2001, p. 19). In this paper we do not consider the effects on market structure, but apply the similar argument that under extension of markets firms have incentives to resist to strengthened competitive pressure resulting in the scale effects that enforce technology growth.

Our model also relates to a simple model of multilateral trade and growth suggested in the abovementioned empirical study by Alberto Alesina et al. (2000, 2004). Transitory growth rate is shown by these authors to depend positively on a linear combination of domestic and global demands with an openness parameter as weight. This intuitive result clarifies the specification of the growth regression on country size and openness referred to above. Growth in the model by Alesina et al., unlike ours, is non-sustainable because of diminishing returns to investment in production capital decided by households, not firms. The reason is that the crucial issue of increasing returns to scale is ignored. However, the main subject of their papers, theory and empirics of endogenous national borders, differs from ours.

3. The model

There are $N \geq 2$ economies trading in the international markets for goods. Production sector in each economy supplies a spectrum of differentiated goods. Each production firm delivers a single good to the market and is engaged in monopolistic competition with other firms. The number of firms is endogenous and determined at each time period, which is sufficiently long for the excess monopoly profits to be eliminated by new entry.

Households in all economies have the same preferences. A household maximizes integral utility subject to an intertemporal budget constraint. International lending and borrowing are permitted, and multilateral trade is balanced over infinite horizon. Labor is the only factor of production and internationally immobile. A household supplies inelastically a unity of labor to the domestic production sector. Countries differ in population size which is time-constant.

3.1 Households

The integral household utility in each country is logarithmic:

$$U_j = \sum_{t=1}^{\infty} \beta^t \ln C_t^j , \tag{1}$$

where β is the discount factor, C_t^j is an aggregate consumption utility index per period t indicating CES preferences over the basket of goods for households in country *j*:

$$C_t^j = \left(\sum_{i=1}^{N} \int_0^{n_{it}} b_{it}(\iota) c_{it}^j(\iota)^{(\sigma-1)/\sigma} d\iota \right)^{\sigma/(\sigma-1)} . \tag{2}$$

Here $c_{it}^j(\iota)$ is consumption of good ι produced in country *i* by household in country *j*, $b_{it}(\iota)$ is the quality weight of this good, n_{it} is the number of goods produced in country *i*, and $\sigma > 1$ is the intratemporal elasticity of substitution across goods. The superscript j relates to a country of destination, while the subscript i relates to a country-supplier.

The intertemporal budget constraint is

$$\sum_{t=1}^{\infty} \beta^t P_t^j C_t^j = \sum_{t=1}^{\infty} \beta^t w_{jt} , \tag{3}$$

where w_{jt} is the household factor income per period, P_t^j is the aggregate dual price index:

$$P_t^j = \left(\sum_{i=1}^{N} \int_0^{n_{it}} b_{it}(\iota)^{\sigma} p_{it}^j(\iota)^{1-\sigma} d\iota \right)^{1/(1-\sigma)} , \tag{4}$$

and $p_{it}^j(\iota)$ is the price of the good exported by firm ι from country *i* to country *j*. There is no capital market in the model, and the flows of future incomes and expenditures in (3) are discounted with parameter β[8].

At any time period households choose a bundle of goods subject to the intratemporal budget constraint:

$$\sum_{i=1}^{N} \int_0^{n_{it}} p_{it}^j(\iota) c_{it}^j(\iota) d\iota = e_t^j , \tag{5}$$

where $e_t^j = P_t^j C_t^j$ is the household expenditure in country *j*.

The international market for goods is common in the basic model with zero trading costs. Under this assumption firms set prices identically for all destinations, $p_{it}^j(\iota) = p_{it}(\iota)$, implying that

[8] Discounting the stream of incomes and expenditures with any time-constant factor below 1 would not alter the model results.

the aggregate consumer price index (CPI) is also the same across countries, $P_t^j = P_t$. In what follows the model is extended to include trading costs.

3.2 Firms

Within any time period competition among firms occurs in three stages. At the first stage the number of firms is determined through entry, exit or merger. At the second stage acting firms compete for market niches by investment in technology that allows them to obtain a monopoly position within the niche. At the third stage, as the new technology has been installed, firms behave in the Dixit-Stiglitz sense: they set monopoly prices and produce.

Firms are symmetric in each country, and firm index ι is omitted in what follows. Production function of a firm in country *i* is $q_{it} = d_{it} l_{it}$ where q_{it} is output, l_{it} is labor input, d_{it} is labor productivity. Production exhibits increasing returns to scale and requires fixed labor input f at any time period.

Firms are seeking to expand revenues through technology investment including expenditures on R&D, product and process innovations, new technology adoption, restructuring, reorganization, on-job training of personnel. All these activities allow a firm to raise its technology level denoted a_{it} and related either to quality of goods indicated by the quality weight of the firm in price index (4)[9], b_{it}^{σ}, or to labor productivity measured as $d_{it}^{\sigma-1}$(and also related to the firm weight in the price index, as will be clear below). For the case of quality competition we let labor productivity be unity in all countries, $d_{it} = 1$, and, for the case of productivity competition, utility weights are assumed equal to unity, $b_{it} = 1$. In both cases firms compete for weights in the price index; such a specification of technology competition is non-standard and important for our inferences.

The net profit of a firm π_{it} is equal to operating profit $\Pi_{it} = r_{it} - (w_{it} / d_{it}) q_{it}$, less investment and fixed production cost: $\pi_{it} = \Pi_{it} - w_{it}(x_{it} + f)$, where $r_{it} = p_{it} q_{it}$ denotes the firm revenue, w_{it} is wage – the factor income of households in country *i*, x_{it} is labor input in technology growth. At any period firms maximize the net present value of profits

$$v_{it} = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \pi_{i\tau} \tag{6}$$

by selecting a sequence of labor inputs, $l_{i\tau}, x_{i\tau} \geq 0$, and making price-output decisions, subject to initial technology level a_{i0} and the production function for technology growth:

$$\Delta a_{it} = (\overline{a}_{it-1} / \varphi_i) x_{it} \tag{7}$$

Here $\Delta a_{it} = a_{it} - a_{it-1}$ is the increase of technology level at time period *t*, factor $\overline{a}_{it-1} / \varphi_i$ indicates productivity of labor input in technology growth x_{it}, and $\overline{a}_{it-1}$ is the average technology level in country *i* at the beginning of period *t*. The factor $\overline{a}_{it-1} / \varphi_i$ refers to a national knowledge spillover: a higher average level of existing technology in the economy makes technology improvement less costly for all local firms.[10]

Technology cost parameter φ_i is assumed to depend on the country size, $\varphi_i = \varphi(\gamma_i)$ indicating a positive country-size effect on the extent and intensity of national knowledge spillovers.[11] This effect is assumed more pronounced for a larger economy that is $\varphi'(\gamma_i) \leq 0$ because of the network externalities facilitating the processes of knowledge dissemination within national borders.

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[9] David Hummels and Peter Klenow (2005, p. 706) measure quality differentiation of goods with utility weights, b_{it} in our notation.

[10] This specification of external economies ensures existence of constant growth rate paths, as is the case in Romer (1990), Rivera-Batiz and Romer (1991), Grossman and Helpman (1991) and some other endogenous growth models.

[11] We, thus, put aside other important structural factors of cross-country variation of growth like quality of institutions and educational level.

In each country the cost of entry-exit-merger is zero. The number of firms adjusts to meet the zero net present value condition $v_{it} = 0$ at any time period, implying the zero net profit condition:

$$\pi_{it} = 0 . \tag{8}$$

A new entrant has free access to the "state of the art" technology of the past period, ensuring the symmetry of firms operating in the economy.

Due to the symmetry of local firms $\overline{a}_{it-1} = a_{it-1}$, and this equilibrium condition is accounted for by a firm in making technology investment decision. Moreover, the firm internalizes symmetric actions of all other local firms by taking into consideration that all of them de facto replicate its own actions. Each firm, thus, takes into account the symmetric reaction of domestic rivals before it solves for the first-order condition, similarly to the consistent conjectures assumption in the models of oligopoly theory (e.g. Bresnahan 1981). As will be shown below, such a time sequencing of investment decisions in our model eliminates technology competition between domestic firms for the market niches. This allows focusing on the pure effects of international trade on incentives of firms to generate growth through competition with foreign rivals.

3.3 Trading equilibrium in the common market

Trading equilibrium path is defined as a solution to the household problem (1)-(5) and the firm problem (6)-(7) satisfying at each time period the zero-profit conditions (8), the market-clearing conditions for goods

$$q_{it} = \sum_{j=1}^{N} \gamma_j c_{it}^{j} , \tag{9}$$

and the national labor markets

$$n_{it}(l_{it} + f + x_{it}) = \gamma_i , \tag{10}$$

$i = 1,..., N$. Here γ_i is the country *i* share in the world population normalized to unity.

The model is closed by a normalizing equation. For logarithmic utility (1) and intertemporal budget constraint (3), the household expenditure is constant over time, $e_t^j = e^j$. A convenient way of normalization is to set the global expenditure to unity:

$$\sum_{j=1}^{N} E^j = 1, \tag{11}$$

where $E^j = \gamma_j e^j$ denotes aggregate expenditure of country *j*. Summing up the household budget constraints per period (5) across countries and accounting for the market-clearing conditions (9) imply, according to the Walras law, the equality of global demand and supply of goods (or of global borrowing and lending) at any period of time, $\sum_i n_{it} r_{it} = \sum_j E^j = 1$. Equivalently, the sum of national incomes across countries is unity:

$$\sum_{i=1}^{N} w_{it} \gamma_i = 1. \tag{12}$$

The market share of the country in global production is denoted s_{it} and equal to the number of firms at home times the firm's revenue: $s_{it} = n_{it} r_{it} / \sum_k n_{kt} r_{kt} = n_{it} r_{it}$.

4. Analysis

At the third stage of any period a firm sets a price according to the constant mark-up rule: $p_{it} = (1+\mu) w_{it} / d_{it}$, where $\mu = 1/(\sigma - 1)$ is a profit margin that defines operating profit as a fraction of revenue, $\Pi_{it} = r_{it} / \sigma$. Combining consumer demand in country *j* faced by the firm from country *i* $c_{it}^j = \left(p_{it} / P_t b_{it}\right)^{-\sigma} C_t^j$ with the intratemporal household budget constraint (5) yields $\sum_i n_{it} (p_{it} / P_t) \left(p_{it} / P_t b_{it}\right)^{-\sigma} = 1$. Hence, the market share of country *i* is $s_{it} = n_{it} b_{it}^{\sigma} (p_{it} / P_t)^{1-\sigma}$ or, substituting for the aggregate price index (4),

$$s_{it} = \frac{n_{it} b_{it}^{\sigma} p_{it}^{1-\sigma}}{\sum_k n_{kt} b_{kt}^{\sigma} p_{kt}^{1-\sigma}}. \qquad (13)$$

For the case of quality competition the technology level is defined by the quality factor, $a_{it} = b_{it}^{\sigma}$. The firm revenue is $r_{it} = s_{it} / n_{it} = b_{it}^{\sigma}(p_{it} / P_t)^{1-\sigma} = a_{it}\left((1+\mu)w_{it} / P_t\right)^{1-\sigma}$ because $d_{it} = 1$ and the firm price is $p_{it} = (1+\mu)w_{it}$. For the case of productivity competition the technology level is measured by the productivity factor: $a_{it} = d_{it}^{\sigma-1}$. Since $b_{it} = 1$ and $p_{it} = (1+\mu)w_{it} / d_{it}$, we have $r_{it} = d_{it}^{\sigma-1}\left((1+\mu)w_{it} / P_t\right)^{1-\sigma} = a_{it}\left((1+\mu)w_{it} / P_t\right)^{1-\sigma}$. In both cases the firm revenue is $r_{it} = a_{it}\left((1+\mu)w_{it} / P_t\right)^{1-\sigma}$ and corresponds to the firm weight in the price index (4).

From (13), the firm revenue can be represented as inverse of the total weighted number of firms across countries

$$r_{it} = 1/\left(n_{it} + \sum_{k \neq i} n_{kt} h_{kit}\right) \qquad (14)$$

where $h_{kit} = r_{kt} / r_{it}$ is *the relative revenue* of a country k firm with respect to a country i firm. It is invariant to specification of technology level in terms of quality weights or productivities:

$$h_{kit} = (a_{kt} / a_{it})(w_{kt} / w_{it})^{1-\sigma}. \qquad (15)$$

At the second stage the firm makes a revenue-shifting investment decision by anticipating the expected revenue per period as (14)-(15). The weight of domestic producers in denominator of (14) is unity because the firm internalizes their symmetry basing on the consistent conjecture that all other domestic firms do the same. This property implies two important consequences. First, the firm revenue (14)-(15) is strictly concave in the technology level a_{it} entailing internal solution for the firm maximization problem (6)-(7) and dramatic simplification of the formal analysis. Second, in the case of autarky there is no reason for local firms to compete with each other for the market niches and, hence, to make investment in technology.[12] Only competition with foreign firms creates incentives to invest generated at any time period by deviations of relative revenues $h_{kit}, k \neq i,$ from unity.[13] By this reason, technology growth in the model is driven by the competitive pressure effect of international trade on firms. This property establishes, in extreme way, the leading role of trade and persistent competition for market niches in technological progress emphasized, for instance, by Allyn Young (see footnote 3).

4.1 Market shares

The equilibrium number of firms in any country is determined at the first stage of any period via the zero-profit condition (8) represented in units of labor as

$$\Pi_{it} / w_{it} - (\varphi_i / a_{it-1}) a_{it} = f - \varphi_i . \qquad (8')$$

The left-hand side can be viewed as labor input generating the firm operating profit less the cost of new technology adoption at time period *t*; the right-hand side is *the net fixed cost* of the firm defined as the fixed labor input in production less the cost of the obsolete technology adoption[14]. The net fixed cost is denoted as $\delta_i = f - \varphi_i$ and assumed positive. Equation $(8')$ defines the labor

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[12] Conditions of autarky equilibrium are obtained from the model with trade by setting n_{jt} to 0 for any country *i*, $j \neq i$. In particular, the aggregate consumption index (2) for the autarky case is $C_t^i = \left(\int_0^{n_{it}} b_{it}(\iota) c_t^i(\iota)^{(\sigma-1)/\sigma} d\iota \right)^{\sigma/(\sigma-1)}$. Households and firms solve their problems (1)-(5) and (6)-(7) under zero-profit conditions (8) and market-clearing conditions for labor (10) and goods: $q_{it} = c_t^i$. The normalizing equation determines the level of aggregate expenditure for each country: letting, without loss of generality, $E^i = \gamma_i$ yields $r_{it} = \gamma_i / n_{it}$. Due to the time sequencing within one period, a firm does not affect the number of firms at home and is unable to shift the revenue by investment in technology. As a result, firms in our model do not invest in autarky.

[13] At first glance, the model radically departs in this respect from the recent literature emphasizing heterogeneous domestic firms and endogenous markups, and focusing on the issues of intraindustry selection and other micro-structural shifts (e.g. Bernard et al. 2003, Melitz & Ottaviano 2005). But as Gianmarco Ottaviano (2007) reports, the within-sector heterogeneity is largely attributed to the gap in competitiveness of exporting and non-exporting firms. Our model can be extended to include two heterogeneous types of domestic firms delivering goods to the domestic market (non-exporters) or to the global market (exporters). These types may differ in the capability to make technology improvement as indicated by (7).

[14] From (7), investment in new technology can be viewed as if the firm purchases the new technology a_{it} at price in labor units $\varphi_i / \bar{a}_{it-1}$ and sells simultaneously the old one a_{it-1} at the same price: $x_{it} = (\varphi_i / \bar{a}_{it-1})(a_{it} - a_{it-1})$

force allocation within a firm between variable production input, new technology creation and net fixed input. It is a dynamic extension of the static zero-profit condition $\Pi_{it} / w_{it} = f$ fulfilled for the case of time-constant technology $a_{it} = a_{it-1}$.

Proposition 1. The equilibrium number of firms is

$$n_{it} = s_{it}^2 / \sigma w_{it} \delta_i . \qquad (16)$$

Labor is the only factor of production rewarded by all national income. The labor-market clearing (10) is fulfilled if the nominal wage is equal to the country income per capita:[15]

$$w_{it} = s_{it} / \gamma_i . \qquad (17)$$

The fraction $1/\sigma$ of national income compensates for total fixed labor inputs and technology investment financed from total operating profits of the economy, while the fraction $1-1/\sigma$ compensates for total variable labor inputs in production.

Combining (16) and (17) implies that the number of firms is proportional to the market share, $n_{it} = s_{it}\gamma_i / \sigma\delta_i$, and the equilibrium revenue of a firm $r_{it} = s_{it} / n_{it}$ is time-constant:

$$r_i = \sigma\delta_i / \gamma_i . \qquad (18)$$

The next proposition determines the market shares of countries as a trading equilibrium outcome.

Proposition 2. The equilibrium market share of the country is

$$s_{it} = \frac{(a_{it} / r_i)^{\mu} \gamma_i}{\sum (a_{kt} / r_k)^{\mu} \gamma_k} . \qquad (19)$$

$= (\varphi_i / a_{it-1}) a_{it} - \varphi_i$. Hence, $\varphi_i = (\varphi_i / \overline{a}_{it-1}) a_{it-1}$ is equivalent of the labor cost of the old technology adoption deducted from the fixed production in (8').

[15] Since $s_{it} = r_{it} n_{it}$ we have from (17) $\gamma_i = r_{it} n_{it} / w_{it} = n_{it}(l_{it} + \Pi_{it} / w_{it}) = n_{it}(l_{it} + f + x_{it})$ due to the zero-profit condition (8'). Note that the labor market equilibrium (17) implies that the aggregate expenditure of the country is equal to the permanent market share, $E^i = (1-\beta)\sum_{t=1}^{\infty} \beta^{t-1} s_{it}$, since, from the intertemporal household budget constraint (3), household expenditure is equal to the permanent income, $e^i = (1-\beta)\sum_{t=1}^{\infty} \beta^{t-1} w_{it}$. If the current market share exceeds the permanent one, the country is currently a net lender, otherwise it is a net borrower.

Multiplying both nominator and denominator of (19) by $P_t/(1+\mu)$ yields $s_{it} = \Phi_{it} a_{it}^{\mu} \gamma_i / \sum \Phi_{kt} a_{kt}^{\mu} \gamma_k$, where $\Phi_{it} = P_t/(1+\mu) r_i^{\mu}$ is called here as *a producer price index* (PPI) of country *i*. The equilibrium wage (17) is $w_{it} = \Phi_{it} a_{it}^{\mu} / \sum \Phi_{kt} a_{kt}^{\mu} \gamma_k$. Applying normalization (11) and identity of global demand and supply of goods (12) we have that $\sum \Phi_{it} a_{it}^{\mu} \gamma_i = \sum w_{it} \gamma_i = 1$, and the domestic market share is

$$s_{it} = \Phi_{it} a_{it}^{\mu} \gamma_i \ . \tag{20}$$

The equilibrium wage is $w_{it} = \Phi_{it} a_{it}^{\mu}$, i.e. the PPI times the level of technology which is equal to the equilibrium output of the country per capita denoted $y_{it} = a_{it}^{\mu}$. In the case of quality change, $a_{it} = b_{it}^{\sigma}$, the output per capita is indicated by the quality level as $y_{it} = b_{it}^{1+\mu}$. In the case of productivity change, $a_{it} = d_{it}^{\sigma-1}$, the output per capita coincides with the labor productivity: $y_{it} = d_{it}$. The equilibrium output y_{it} has a broader meaning than the output per worker defined by the firm production function $q_{it} = d_{it} l_{it}$. The former measures the quality of goods, as well as the labor productivity represented by the latter, and is attributed to all workers including those engaged in fixed inputs and technology improvement.

4.2 Firm size and country size

We have obtained that the ratio of wage to output per capita is determined in trading equilibrium by the PPI, which is inversely related to the firm size measured as revenue (18). To meet this size, the firm sets the mark-up price according to the inverse demand schedule, $p_{it} = (1+\mu)\Phi_{it} a_{it}^{\mu} = P_t (a_{it}/r_i)^{\mu}$. The larger is the firm size, the lower are the PPI, the wage level, and the firm price. Thus, for the common-market model, any cross-country variation of technology-adjusted firm price is determined by the variation of firm size as $p_{it} / a_{it}^{\mu} = r_i^{-\mu} P_t$.

The firm size $r_i = \sigma\delta_i / \gamma_i$ is proportional to the net fixed cost per capita revealing two opposite effects of the country size. The first one is the country-size external effect on the technology cost φ_i implying that the net fixed cost is increasing in γ_i: $\delta_i \equiv \delta(\gamma_i) = f - \varphi(\gamma_i)$, $\delta'(\gamma_i) \geq 0$. The second effect indicates the global market pressure on the firm size, which is larger for a smaller country, other things being the same. A firm in this country is forced to set lower technology-adjusted price thus obtaining a competitive advantage through the scale of production.

Counteraction of these effects defines the firm size as a function of country size: $r_i \equiv r(\gamma_i) = \sigma\delta(\gamma_i) / \gamma_i$. It may be decreasing, increasing, or have the minimum point, as shown in figure 3.[16] In the first case the national externality is weak and dominated by the competitive pressure effect of country size: $r_i' \leq 0$ or $-\varphi_i' \leq r_i / \sigma$. In the second case the externality is strong implying that $r_i' > 0$. In the third case the externality is moderate and the firm size curve $r(\gamma_i)$ is U-shaped. The country-size external effect outweighs competitive pressure for a large economy but is outweighed by this pressure for a small one. In this case the firm size is relatively high both for the large and small economies due to the dominating effect of either competitive pressure or country-size externality, respectively.

The firm size measured as the number of employees per firm is equal to

$$\gamma_i / n_{it} = \sigma\delta_i / s_{it}, \tag{21}$$

(since $n_{it} = s_{it} / r_i$) and inversely related to the domestic market share. Consequently, the global market pressure on a small or less advanced economy enforces the local firm to increase the number of employees.[17]

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[16] For example, the firm size curve $r(\gamma_i)$ is decreasing (increasing) for the technology cost parameter given as $\varphi_i = f - \gamma_i^{\chi}$, $0 < \chi < 1$ ($\chi > 1$), and U-shaped for $\varphi_i = f - e^{\chi\gamma_i}$ with minimum point $\gamma_i = 1/\sigma\chi$. In the special case $\varphi_i = f - \chi\gamma_i$ the firm size is the same for all countries: $r_i = \sigma\chi$.

[17] For comparison, the equilibrium number of firms in the Krugman (1980) model of bilateral trade between countries that differ in population size is $n_{it} = \gamma_i / f\sigma$ (eq.11, p. 952, with notation of our paper) implying that labor input by

Figure 3 here

4.3 Growth

The factor prices and national incomes are determined at any period together with the technology growth rates. From (18), the firm operating profit is proportional to the firm size, $\Pi_i = r_i / \sigma = \delta_i / \gamma_i$. Due to (17), the profit in labor units is $\Pi_i / w_{it} = \delta_i / s_{it}$ indicating labor resources available for technology investment and fixed production inputs by the firm. Inserting this into the zero-profit condition ($8'$) yields straightforwardly the equilibrium growth rate of technology.

Proposition 3. The equilibrium technology growth satisfies:

$$(\varphi_i / a_{it-1})a_{it} = (1 - s_{it})\delta_i / s_{it} \tag{22}$$

The labor cost of technology a_{it} adoption is thus equal to the fraction $1 - s_{It}$ of the firm's profit in labor units δ_i / s_i. Proposition 3 implies that the cross-country variation of technology growth is defined by two factors: the country-size externality and the market size indicating competitive pressure on the firm. Indeed, equation (22) can be rewritten as:

$$a_{it} / a_{it-1} = g(\gamma_i)(1/ s_{it} - 1) \tag{22$'$}$$

where $g(\gamma_i) \equiv \delta(\gamma_i)/\varphi(\gamma_i)$ is the relative net fixed cost as an increasing function of country size: $g'(\gamma_i) = (\delta_i / \varphi_i)' = (f / \varphi_i)' \; = -f\varphi_i' / \varphi_i^2 \geq 0$. We call $g(\gamma_i)$ as *the country-size factor* of growth and $(1/ s_{it} - 1)$ as the *market-size factor*. The former relates to the national externality, and the latter is the inverse of the relative national income capturing the competitive pressure effect of the global market on technology growth.

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the firm in the Krugman model is $\gamma_i / n_{it} = f\sigma$. The number of employees per firm in our model is defined by the ratio δ_i / s_{it} instead of the fixed production cost f.

The system of equations (19), (22′) determines global dynamics in terms of market shares and technology levels of countries. The market-size factor is the key one for stabilizing global dynamics: if s_{it} is small, the country grows rapidly and vice versa.[18] According to (22′), the growth of equilibrium output per capita is $y_{it}/y_{it-1} = \left(g(\gamma_i)(1/s_{it} - 1)\right)^{\mu}$ because $y_{it} = a_{it}^{\mu}$ and it is positive, $y_{it} > y_{it-1}$, if $s_{it} < g(\gamma_i)/(1+g(\gamma_i)) = \delta_i / f$. This condition is supposed to hold for all countries at initial period of time implying that countries do not differ much in population size and initial technology level. The condition of positive growth defines a domain in the hyperplane of market shares $\sum_i s_{it} = 1$ depicted for the case of two countries as interval *AB* in figure 4. This interval is non-empty if technology costs in both countries are sufficiently low: $\varphi_1 + \varphi_2 < f$.

Figure 4 here

4.4 The stationary growth path

All countries grow at the same constant rate along the stationary balanced growth path. The stationary distribution of national incomes is calculated from (22′):

$$s_i = \frac{g(\gamma_i)}{g(\gamma_i) + g} \tag{23}$$

where $g = a_{it} / a_{it-1}$ is the stationary growth rate solving equation:[19]

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[18] Noteworthy, our model does not predict, unlike the early endogenous growth models, that the technology growth rate is unboundedly increasing in time because of population growth. Neither predicts it that balanced growth is positive only if population in a country grows, as is the case for the quasi-endogenous growth models suggested by Charles Jones (1995) to overcome this deficiency of the endogenous growth theory. If we introduce, for example, a population growth rate $\nu > 1$, the same for all countries, then the global demand will grow at this rate, $\nu^t \sum E^j = \nu^t$, and the number of firms in each country will be $n_{it} = s_{it}\nu^t\gamma_i / \sigma\delta_i$. The firm size and profit in labor units will be the same as in the model with constant population, $r_{it} = s_{it}\nu^t / n_{it} = \sigma\delta_i / \gamma_i$ and $\Pi_{it} / w_{it} = \delta_i / s_{it}$, implying that the market-size factor of growth in (22′) does not change. The country-size factor $g(\gamma_i \nu^t)$ will be increasing in time with population size but tending to a constant under a proper specification of the country-size externality as defined by $\varphi(\gamma_i \nu^t)$. Consequently, growth in our model is positive under constant population and does not increase unboundedly under population growth.

[19] Equality of global supply and demand (12) does not hold automatically for the stationary growth path, because the steady-state market shares (23) are calculated from growth equations (22′) under presumption that growth is the same

$$\sum \frac{g(\gamma_j)}{g(\gamma_j)+g} = 1 . \tag{24}$$

In the special case of symmetric country size, $\gamma_i = 1/N$, the steady-state market shares are $s_i = 1/N$, and the global technology growth is $g = g(1/N)(N-1)$.[20] In the other special case of bilateral trade between asymmetric countries the steady-state income distribution is $s_1 = \rho^{1/2}/(1+\rho^{1/2})$, $s_2 = 1/(1+\rho^{1/2})$ where $\rho = g(\gamma_1)/g(\gamma_2)$ is the relative country-size factor of growth. Equation (24) implies that the latter is equal to $g = \left(g(\gamma_1)g(\gamma_2)\right)^{1/2}$.

Generally, the global balanced growth rate solving (24) is a monotone increasing and homogenous degree of one function of the array of the country-size factors: $(g(\gamma_i))_{i=1}^{N}$. Growth is positive, $g > 1$, only if the following condition holds: $\sum g(\gamma_i)/(g(\gamma_i)+1) = \sum \delta_i/(\delta_i + \varphi_i) > 1$, implying $\sum \delta_i > f$ or $\sum \varphi_i < f(N-1)$. In other words, total technology cost across countries must be below the fixed production cost times $N-1$.

The balanced growth path is globally stable for the two-country case. Indeed, (19) implies $s_{1t}/s_{2t} = (a_{1t}/a_{2t})^{\mu}(r_2/r_1)^{\mu}\gamma_1/\gamma_2$ or $a_{1t}/a_{2t} = \xi_t^{\sigma-1}(r_1/r_2)(\gamma_2/\gamma_1)^{1-\sigma}$, where $\xi_t = s_{1t}/s_{2t}$ is the relative income of country 1. Inserting this into (22′) yields the scalar difference equation for the relative national income: $(\xi_t/\xi_{t-1})^{\sigma-1} = (g(\gamma_1)/g(\gamma_2))\xi_t^{-2}$, which is globally stable since $\xi_t = \rho^{\frac{1}{\sigma+1}}\xi_{t-1}^{\frac{\sigma-1}{\sigma+1}}$.[21] Any trajectory starting within the domain of positive growth converges to the stationary growth path, as depicted in figure 4. This property results from the negative relation

across countries. Market shares, therefore, do not equal in total unity for an arbitrary stationary growth rate *g*, which is defined to ensure trade balance in the global economy, as required by (24).

[20] Note that the model assumption that a firm internalizes the symmetry of domestic producers is somewhat *ad hoc* for the case of symmetric steady-state path. Indeed, why the firm does not internalize the global symmetry of producers in this situation? We can suggest that the case that the global economy initially locates *exactly* on the symmetric steady-state path is negligible, while our assumption is not *ad hoc* for any arbitrary close trajectory.

[21] We devote a different paper dealing with the same model to examine the stabilizing effects of trade on global growth and convergence of incomes to the steady-state distribution. Generally, the $2N-1$ dimensional system (19), (22′) for the array of variables s_{it} and a_{it} is globally stable. Daron Acemoglu and Jaume Ventura (2002) have

between the growth rate of technology and the domestic market share (22′), on the one hand, and the positive relation between the market share and the technology level (19), on the other hand. A less developed or smaller economy grows faster implying convergence of national incomes to the stationary distribution. The latter is determined by the cross-country distribution of population size, which is decisive for national knowledge spillovers, rather than by the initial technology disparity eliminated through trade.

5. Trading costs

Thus far we have considered the model with free trade and common global market. In this section the model is extended to introduce a formal definition of the market extent and to examine the effects of openness and country size on growth. Trading costs transform the global common market for goods into *N* interrelated local markets. These costs relate to geographic distances or other trade barriers and are of iceberg type: a fraction of exports is lost in transit. Delivery of goods from country *i* to country *j* raises variable production costs by factor $\tau_{ij} \geq 1$. Without loss of generality, goods are supplied to the home market with no trading costs, $\tau_{ii} = 1$.

5.1. Local and global market shares

Under the trading costs the producer prices p_{it}^{j} differ across countries of destination *j*, as well as the consumer price indices P_t^j. The normalizing equation is identical to (11), $\sum E^j = 1$, and the equality of total demand and supply entails, similarly to (12), that $\sum w_{it}\gamma_i = 1$.

A household in country *j* demands $c_{it}^{j} = \left(p_{it}^{j} / P_t^j b_{it}\right)^{-\sigma} C_t^j$ units of goods supplied by a firm from country *i*. The intratemporal household budget constraint (5) is

demonstrated that trade in goods can lead to a stable world income distribution, because faster than average accumulation of capital results in terms-of-trade deterioration implying de facto diminishing returns and convergence.

$\sum_i n_{it}(p_{it}^j / P_t^j)(p_{it}^j / P_t^j b_{it})^{-\sigma} = 1$ implying *the local market share* of producing country *i* in country of destination *j* as

$$s_{it}^j = n_{it} b_{it}^{\sigma} (p_{it}^j / P_t^j)^{1-\sigma}. \tag{25}$$

The local market revenue of the firm is $r_{it}^j = s_{it}^j E^j / n_{it} = b_{it}^{\sigma} (p_{it}^j / P_t^j)^{1-\sigma} E^j$ or, substituting for P_t^j and rearranging terms,

$$r_{it}^j = E^j / \left(n_{it} + \sum_{k \neq i} n_{kt} h_{kit}^j\right) \tag{26}$$

where $h_{kit}^j = r_{kt}^j / r_{it}^j = b_k^{\sigma} p_{kt}^{j1-\sigma} / b_{it}^{\sigma} p_{it}^{j1-\sigma}$ is *the relative revenue* of a country *k* firm competing with a country *i* firm in the local market *j*.

The firm sets the price mark-up compensating for the shipping costs: $p_{it}^j = (1+\mu)\tau_{ij} w_{it} / d_{it}$. The relative revenue is written as

$$h_{kit}^j = (a_{kt} / a_{it})(\tau_{kj} w_{kt} / \tau_{ij} w_{it})^{1-\sigma}, \tag{27}$$

where, as above, $a_{it} = b_{it}^{\sigma}$ or $d_{it}^{\sigma-1}$ indicating quality or productivity.

The global revenue of the firm is the sum of local market revenues $r_{it} = \sum_j r_{it}^j$. Define *the global market share* of country *i* as the weighted average of local market shares: $s_{it} = r_{it} n_{it} = \sum_j E^j s_{it}^j$, and *the market extent* of the country as the ratio of squared global market share to the weighted average of squared local market shares, $M_{it} = s_{it}^2 / \sum_j E^j s_{it}^{j2}$.[22] Taking into account (25) and the mark-up pricing rule, the market extent is represented as a function of the consumer price indices as

$$M_{it} = \frac{\left(\sum_j E^j (P_t^j / \tau_{ij})^{\sigma-1}\right)^2}{\sum_j E^j \left(P_t^j / \tau_{ij}\right)^{2(\sigma-1)}} \tag{28}$$

[22] The denominator of this indicator resembles the popular *Herfindahl-Hirschman* index of concentration, but this analogy is wrong since market shares s_{it}^j relate to different local markets *j*, and, generally, $\sum_j s_{it}^j \neq 1$.

Three propositions that follow characterize trading equilibrium in the world with trading costs and generalize propositions 1-3.

Proposition 4. The equilibrium number of firms is

$$n_{it} = s_{it}^2 / \sigma w_{it} \delta_i M_{it} \tag{29}$$

As in the common market case, the equilibrium wage is the ratio of the global market share of the country to the population share, similar to (17), $w_{it} = s_{it} / \gamma_i$. From this and (29), the equilibrium number of firms is $n_{it} = s_{it}\gamma_i / \delta_i M_{it}\sigma$, and the global revenue of the firm is

$$r_{it} = s_{it} / n_{it} = M_{it}\sigma\delta_i / \gamma_i . \tag{30}$$

This is the product of the common market revenue defined by (18) as $r_i = \sigma\delta_i / \gamma_i$ and the market extent of the country M_{it}. The firm size as the number of employees is $\gamma_i / n_{it} = M_{it}\sigma\delta_i / s_{it}$ extending (21). Consistently with empirical regularities discussed in the introduction, the firm size is positively related to the market extent measured as M_{it}. The meaning and properties of this indicator are discussed in what follows.

Trading equilibrium of the model with trading costs is characterized by three kinds of aggregate price indices that differ across countries. The two of these are the consumer price index (CPI) P_t^i and the producer price index (PPI) Φ_{it} introduced above for the common market, and the third one is a *sales price index* (SPI) F_{it} defined below. The next proposition determines the global market shares and the global system of price indices.

Proposition 5. The global market shares of countries are

$$s_{it} = \Phi_{it} a_{it}^{\mu} \gamma_i , \tag{31}$$

and the aggregate price indices Φ_{it}, P_t^i, F_{it} *satisfy*

$$\Phi_{it}^{\sigma-1} = \frac{\sum_j E^j (P_t^j / \tau_{ij})^{2(\sigma-1)}}{F_{it}^{\sigma-1}(1+\mu)^{\sigma-1} r_i} \tag{32}$$

$$F_{it}^{\sigma-1} = \sum_j E^j (P_t^j / \tau_{ij})^{\sigma-1} \tag{33}$$

$$P_t^{i1-\sigma} = \sum_k s_{kt} (F_{kt} \tau_{ki})^{1-\sigma} \tag{34}$$

$i, j, k = 1, ..., N$.

Equation (31) extends the market share equation (20) obtained for the common market case. The national output is evaluated through the PPI Φ_{it} which is determined together with the consumer and sales price indices P_t^i and F_{it} by equations (32)-(34). According to (32), the transformed PPI $\Phi_{it}^{\sigma-1}$ is proportional to the ratio of weighted squared and trading-cost-adjusted CPI, $\sum_j E^j (P_t^j / \tau_{ij})^{2(\sigma-1)}$, to the transformed SPI $F_{it}^{\sigma-1}$. This ratio turns into the common price index $P_t^{\sigma-1}$ in the case of no trade barriers, $\tau_{ij} = 1$. The SPI $F_{it}^{\sigma-1}$ is defined by (33) as the average of trading-cost-adjusted CPI of countries of destinations weighted with their shares in global demand (and is also equal to $P_t^{\sigma-1}$ under zero trading costs). Finally, the CPI P_t^i is the CES price aggregate of trading-cost-adjusted SPI, $F_{kt}\tau_{ki}$, weighted with shares of countries in global production (34). Proposition 5 implies, similarly to proposition 2, that equilibrium wage is equal to equilibrium output per capita $y_{it} = a_{it}^{\mu}$ times the PPI of the country, $w_{it} = \Phi_{it} a_{it}^{\mu}$.

Proposition 5 implies that the equilibrium local market shares of country *i* are[23]

$$s_{it}^j = s_{it} \left(P_t^j / \tau_{ij} F_{it} \right)^{\sigma-1}. \tag{35}$$

The local market share relates to the global one as the ratio of trading-cost adjusted CPI of country of destination $j, P_t^j / \tau_{ij}$, to the SPI of producing country *i*, F_{it}. Dividing both parts of the CPI equations (34) by $P_t^{i\sigma-1}$ and using (35) yields *N* market-clearing conditions for all local markets *j*

[23] Inserting successively (29), (17), (31) into (25) yields $s_{it}^j = n_{it} b_{it}^{\sigma} (p_{it}^j / P_t^j)^{1-\sigma}$ $= (s_{it}^2 / \sigma w_{it} \delta_i M_{it}) a_{it} ((1+\mu)\tau_{ij} w_{it})^{1-\sigma} P_t^{j\sigma-1}$ $= (s_{it}\gamma_i / \delta_i M_{it} \sigma) a_{it} ((1+\mu)\Phi_{it} a_{it}^{\mu})^{1-\sigma} (P_t^j / \tau_{ij})^{\sigma-1}$ $= s_{it} ((1+\mu)^{\sigma-1} \Phi_{it}^{\sigma-1} r_i M_{it})^{-1} (P_t^j / \tau_{ij})^{\sigma-1}$. From (28), (32), (33) we have $(1+\mu)^{\sigma-1} \Phi_{it}^{\sigma-1} r_i = F_{it}^{\sigma-1} / M_{it}$. Consequently, $s_{it}^j = s_{it} (P_t^j / \tau_{ij} F_{it})^{\sigma-1}$.

$$\sum_i s_{it}^j = 1 \tag{36}$$

One of these equations is abundant by the Walras law: multiplying both parts of (36) by E^j and summing up over countries entails equality of global supply and demand (12), $\sum s_{it} = 1$. The local market-clearing equations (36) hence define, equivalently to (34), an array of $N-1$ relative CPI.

5.2. Growth and the market extent

From (30), the firm operating profit is equal to the product of the net fixed cost per capita δ_i / γ_i and the market extent, $\Pi_{it} = M_{it}\delta_i / \gamma_i$, or, in units of labor, $\Pi_i / w_{it} = M_{it}\delta_i / s_{it}$. Inserting this into the zero-profit condition ($8'$) and slightly rearranging terms yields the main assertion of our paper.

Proposition 6 (a modified "Adam Smith's theorem"). The equilibrium growth rate of technology is limited by the extent of the market:

$$a_{it} / a_{it-1} = g(\gamma_i)(M_{it} / s_{it} - 1). \tag{37}$$

Growth equation (37) extends (22′) to the trading costs case. As above, the growth rate depends on the country-size and market-size factors. The former, $g(\gamma_i)$, is the same as above, while the latter is adjusted with the market extent M_{it} indicating the degree of the firm's openness to global trade relative to the common market case. The growth rate of equilibrium output per capita (household income net-off changes of PPI Φ_{it}) is $y_{it} / y_{it-1} = \left(g(\gamma_i)(M_{it} / s_{it} - 1)\right)^{\mu}$. It is positive, if $s_{it} < M_{it} g(\gamma_i)/(1 + g(\gamma_i)) = M_{it}\delta(\gamma_i)/f$, that is countries do not differ essentially in the global market shares and are quite open to trade.

Proposition 6 states that the market extent constrains the market-size factor of growth. The maximal value of this indicator is unity (since $s_{it}^2 \le \sum_j E^j s_{it}^{j2}$) and corresponds to the common market case for which the local market shares of country i are equalized across all countries: $M_{it} = \left(\sum E^j\right)^2 / \sum E^j = 1$.[24] The opposite extreme case is an autarkic economy with infinite trading costs τ_{ij} and τ_{ji} for $j \neq i$. If country i is in autarky, then the local market share at home s_{it}^i is equal to one, the local market shares overseas $s_{it}^j, j \neq i,$ are zero, and the market extent coincides with the domestic aggregate demand, $M_{it} = E^i$ (as has been shown above, there is no growth in autarky, and this case is pointed here to clarify the meaning of M_{it}).

Any deviation of M_{it} from unity is caused by variation of local market shares due to the trading costs. Normally, the local market shares overseas deviate downwards from the global one, $s_{it}^j < s_{it}$ for $j \neq i$, and the local market share at home deviates upwards, $s_{it}^i > s_{it}$. In this case an increase of the local market share abroad enhances the market extent, while an increase of this share at home has an opposite effect. As clear from (35), the local market share overseas is below the global one if $P_t^j / \tau_{ij} < F_{it}$, or the trading-cost adjusted CPI of country of destination exceeds the SPI at home (intuitively, this may not be the case if the domestic economy is quite small and open to trade). Similarly, the local market share at home exceeds the global one if $P_t^i > F_{it}$ (this is the case for a large economy under barriers to trade).

The market extent of the country depends on the global structure of trading costs. For instance, condition that country i faces no barriers in trade with all countries, $\tau_{ij} = \tau_{ji} = 1$ for all destinations j, is only necessary for the market extent of country i to be maximal. The sufficient condition is that price indices P_t^j coincide for all economies. To clarify this, consider an example of three countries such that trade between countries 1 and 2 is costly, $\tau_{12}, \tau_{21} > 1$, and between each of these and country 3 is free, $\tau_{3j} = \tau_{j3} = 1$, $j = 1{,}2$. The CPI vary for these countries and, from

[24] The local market shares providing maximum to $M_{it} = s_{it}^2 / \sum_j E^j s_{it}^{j2}$ satisfy the necessary and sufficient

(35), the local market shares of country 3 deviate from the global one because $P_t^j \neq F_{3t}$ implying that $s_{3t}^j \neq s_{3t}$. The market extent of country 3 is, hence, below unity, although the trading costs for this country are zero. The costs of bilateral trade between countries 1 and 2 affect the market extent of country 3 indirectly, through cross-country variation in price levels.

6. The stationary growth path and the real exchange rates

The complete system for global dynamics consists of $6N-1$ highly nonlinear equations (28), (31)-(34), (37) for market shares, technology levels, market extents and the triplet of aggregate price indices. Such a system is hardly tractable analytically, even for the two-country case. We therefore focus on the stationary growth path along which technologies in all countries grow at the same constant rate g satisfying the steady-state version of (37):

$$g = g(\gamma_i)(M_i / s_i - 1) \tag{38}$$

The time subscript is omitted here and henceforth. The household budget constraint (3) implies that the aggregate consumer expenditure coincides with the stationary market share, $E_i = s_i$. Combining (33) with (34), rearranging terms and taking into account (28), (38), (12) yields a $3N-1$ dimensional system for the array of N steady-state market extents M_i, $N-1$ relative CPI or *the real exchange rates* $\psi_i = (P^i / P^N)^{\sigma-1}$, $N-1$ global market shares s_i and global growth rate g:

$$M_i = \frac{\left(\sum_j s_j \psi_j \vartheta_{ij}\right)^2}{\sum_j s_j (\psi_j \vartheta_{ij})^2}, \tag{39}$$

$$\psi_i \sum_k \frac{s_k \vartheta_{ki}}{\sum_j \psi_j s_j \vartheta_{ij}} = 1, \tag{40}$$

$$g(\gamma_i)(M_i / s_i - 1) = g(\gamma_j)(M_j / s_j - 1), \tag{41}$$

---

conditions: $\partial M_{it} / \partial s_{it}^j = 2E^j (s_{it} - s_{it}^j M_{it}) / \sum_j E^j s_{it}^{j2} = 0$ implying $s_{it}^j = s_{it} / M_{it} = \sum_j E^j s_{it}^{j2} / s_{it} = s_{it}$ for all *j*.

$$\sum s_i = 1, \tag{42}$$

where $\vartheta_{ij} = \tau_{ij}^{1-\sigma} \leq 1$ indicates the degree of *openness for the trade flow* from country i to country j as a fraction of firms' potential revenues retained in shipping. This system consists of N market extent equations (39), the steady-state versions of (28), N market-clearing equations (40), the steady-state versions of (34) or (36) that determine the real exchange rates, $N-1$ conditions of growth rate equalization (41), and the global trade balance (42) which determines the stationary growth rate, similarly to (24). By the Walras law, one of the local market-clearing equations (40) is abundant, and the total number of equations in (39)-(42) is $3N-1$.

The steady-state global market shares are defined from the growth equation (38) as

$$s_i = \frac{M_i g(\gamma_i)}{g + g(\gamma_i)}. \tag{43}$$

The market share is increasing in the country size and the market extent. The total balance (42) determines the stationary growth rate as a monotonously increasing function of the array of the market extents, $g = g(M_1,...,M_N)$. The stationary growth is positive, $g > 1$, only if $\sum \frac{M_i g(\gamma_i)}{1+g(\gamma_i)} = \sum \frac{M_i \delta_i}{\delta_i + \varphi_i} > 1$ or, since $\delta_i / (\delta_i + \varphi_i) = \delta_i / f$, the weighted sum of technology costs across countries is quite small, $\sum M_i \varphi_i < f\left(\sum M_i - 1\right)$. Alternatively, the weighted sum of the market extents must be sufficiently high, $\sum M_i \delta_i > f$. The shaded triangle in figure 5 depicts the domain of positive stationary growth for $N = 2$.

The examples that follow illustrate the solution of (39)-(42) and demonstrate how the openness of trade flows and the relative country size affect the market-size factor of growth.

Figure 5 here

6.1. N symmetric countries

Consider the case of N countries with equal population size $\gamma_i = 1/N$ and symmetric trading costs such that the degrees of trade flow openness are $\vartheta_{ij} = \vartheta$ and $\vartheta_{ii} = 1$. The steady-state price level is the same across countries, and the real exchange rate is unity, $\psi_i = 1$ for all i. The market share and the market extent are also the same: $s_i = 1/N$, and, from (39),

$$M_i = \frac{\left(s_i + (1-s_i)\vartheta\right)^2}{s_i + (1-s_i)\vartheta^2} = \frac{(1+(N-1)\vartheta)^2}{N(1+(N-1)\vartheta^2)} \equiv M^{(N)}(\vartheta) \, . \tag{44}$$

For the extreme case of free trade $\vartheta = 1$ and $M^{(N)}(1) = 1$. Formally, for the other extreme case of overall autarky $\vartheta = 0$, $M^{(N)}(0) = 1/N$ coinciding with the market share s_i or aggregate domestic consumption E_i. The market extent (44) is increasing in trade flows openness: $\frac{\partial M^{(N)}(\vartheta)}{\partial \vartheta} = \frac{2(1-\vartheta)(N-1)(1+(N-1)\vartheta)}{(1+(N-1)\vartheta^2)^2} > 0$. The stationary growth rate is $g = g(1/N)(NM^{(N)}(\vartheta) - 1)$. In the next example we use the symmetric market extent for two countries: $M^{(2)}(\vartheta) = \frac{(1+\vartheta)^2}{2(1+\vartheta^2)}$.

6.2. Two asymmetric countries

Consider two countries 1 and 2 with different size and symmetric trading costs such that $\vartheta_{12} = \vartheta_{21} = \vartheta$. Let $\psi = (P^1/P^2)^{\sigma-1}$ denote the real exchange rate of country 1 in units of relative revenue. As shown in appendix B, the global market shares satisfying market-clearing equations (40) are

$$s_1 = \frac{1-\vartheta\psi}{1-\vartheta\psi + (\psi-\vartheta)\psi}, \qquad s_2 = \frac{(\psi-\vartheta)\psi}{1-\vartheta\psi + (\psi-\vartheta)\psi} \, . \tag{45}$$

These coincide with (43) for the equilibrium real exchange rate derived below. The market share (45) is decreasing in this rate[25], and $s_i = 1/2$ in the symmetric case $\psi = 1$.[26]

As is also shown in appendix B, the growth equation (38) is represented for country 1 as $g = g(\gamma_1)\left(\dfrac{(1-\vartheta^2)^2}{H(\psi,\vartheta)} - 1\right)$, where $H(\psi,\vartheta) = (1-\vartheta\psi)^2 + (1-\vartheta\psi)(1-\vartheta/\psi)\vartheta^2$. Similarly, for country 2 $g = g(\gamma_2)\left(\dfrac{(1-\vartheta^2)^2}{H(1/\psi,\vartheta)} - 1\right)$, where $H(1/\psi,\vartheta) = (1-\vartheta/\psi)^2 + (1-\vartheta\psi)(1-\vartheta/\psi)\vartheta^2$. Equalization of these growth rates (41) yields the real exchange rate equation:

$$\rho\left(\frac{(1-\vartheta^2)^2}{H(\psi,\vartheta)} - 1\right) = \frac{(1-\vartheta^2)^2}{H(1/\psi,\vartheta)} - 1, \tag{46}$$

where, as above, $\rho = g(\gamma_1)/g(\gamma_2)$ is the relative country-size factor of growth of country 1 indicating the size difference between the countries. The global market shares are positive for $\psi \in (\vartheta, 1/\vartheta)$, as seen from (45). The left-hand side of (46) is increasing in ψ on this interval, equal to zero for $\psi = \vartheta$, and has vertical asymptote $\psi = 1/\vartheta$. The right-hand side is decreasing, has vertical asymptote $\psi = \vartheta$, and equal to zero for $\psi = 1/\vartheta$. Figure 6 demonstrates that equation (46) has unique solution denoted $\psi(\vartheta,\rho)$. Curves $\tilde{g}_1(\psi)$ and $\tilde{g}_2(\psi)$ in the figure correspond to the left- and right-hand sides of (46), respectively.

Figure 6 here

The steady-state real exchange rate $\psi(\vartheta,\rho)$ is defined by the exogenous parameters of trade flow openness and relative country size. It is unity for size-symmetric countries or zero trading costs: $\psi(\vartheta,1) = \psi(1,\rho) = 1$. Indeed, for $\rho = 1$ (46) implies $H(\psi,\vartheta) = H(1/\psi,\vartheta)$ which is

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[25] $\dfrac{\partial s_1}{\partial \psi} = -\dfrac{\vartheta(1+\psi^2 - 2\vartheta\psi) + 2(\psi-\vartheta)(1-\vartheta\psi)}{(1+\psi^2-2\vartheta\psi)^2} < 0$ for $\vartheta < \psi < 1/\vartheta$.

fulfilled only for $\psi = 1$. For ϑ tending to 1 equilibrium ψ also tends to 1 since it belongs to interval $(\vartheta, 1/\vartheta)$. Consequently, the steady-state real exchange rate deviates from 1, only if the countries differ in size and face barriers to trade.

Figure 7 demonstrates the simulated shape of $\psi(\vartheta, \rho)$ as a solution of (46) belonging to the admissible interval $(\vartheta, 1/\vartheta)$. The real exchange rate is below 1 for a larger economy, $\psi(\vartheta, \rho) < 1$, $\rho > 1$, and above 1 for a smaller economy, $\psi(\vartheta, \rho) > 1$, $\rho < 1$. It is increasing in openness for the former and decreasing for the latter: as seen from figure 7, $\partial\psi / \partial\vartheta > 0$ for $\rho > 1$ and $\partial\psi / \partial\vartheta < 0$ for $\rho < 1$. The real exchange rate is decreasing in the relative country size, $\partial\psi / \partial\rho < 0$.

Figure 7 here

The next proposition establishes a close connection between the model prediction and the empirical evidence on country size and openness as determinants of long-term growth pointed out at the very beginning of this article.

Proposition 7. For the case of two countries with relative size close to 1 the stationary growth rate of country 1 is[27]

$$g = g(\gamma_1)\left[2M^{(2)}(\vartheta) - \frac{\vartheta(\rho - 1)}{1 + \vartheta^2} - 1\right]. \qquad (47)$$

For the symmetric case $\rho = 1$ (47) coincides with symmetric growth rate $g = g(1/2)\left(2M^{(2)}(\vartheta) - 1\right)$ obtained above[28]. For $\rho \neq 1$ the market-size factor deviates from the

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[26] For a small deviation from this case we have $\psi + 1/\psi \approx 2$, and $s_1 = \dfrac{\psi^{-1} - \vartheta}{\psi^{-1} - \vartheta + \psi - \vartheta} \approx \dfrac{1}{2}\dfrac{\psi^{-1} - \vartheta}{1 - \vartheta}$. Similarly, $s_2 \approx \dfrac{1}{2}\dfrac{\psi - \vartheta}{1 - \vartheta}$.

[27] By symmetry of (46), the growth rate of country 2 is $g = g(\gamma_2)\left[2M^{(2)}(\vartheta) - \dfrac{\vartheta(1/\rho - 1)}{1 + \vartheta^2} - 1\right]$.

[28] For the common market case ($\vartheta = 1, M^{(2)}(1) = 1$) equation (47) implies $g = g(\gamma_1)\left(2 - (\rho - 1)/2 - 1\right)$ $= g(\gamma_1)(3 - \rho)/2$. To check correctness of our calculation, the same approximation is obtained for the example of

symmetric case by term $\frac{-\vartheta}{1+\vartheta^2}(\rho-1)$. The latter is positive for the smaller economy, negative for the larger one, and decreasing in relative country size. The effect of trade flow openness on growth is positive (since $\left(\vartheta/(1+\vartheta^2)\right)' = (1-\vartheta^2)/(1+\vartheta^2)^2 > 0$ for $\vartheta < 1$) and diminishing with relative country size.

The key property of the real exchange rate function $\psi(\vartheta,\rho)$ behind this inference is that it is decreasing in the relative country size (as shown in the proof of proposition 7, $\partial\psi/\partial\rho = (\vartheta-1)/4M^{(2)}(\vartheta) < 0$). The market-size factor of growth is, in turn, increasing in the real exchange rate. The smaller economy is, therefore, characterized by a higher real exchange rate resulting in a higher market-size factor of growth, as implied from proposition 7.

7. Discussion of results

The presented model of trade-driven growth is designed to examine the market-size effects on incentives of firms to invest in technology improvement. The growth rate is determined in global trading equilibrium, along with the cross-country distribution of incomes and trade flows and the global structure of price indices. The latter underlie generalization of the basic model with the common market to the world economy model with fragmented national markets, arbitrary trading costs structure and country-size asymmetry. In both cases the global market share of the country specifies the factor price as the producer price index multiplied by the equilibrium output per capita which is defined by the technology level of this country.

The growth rate obtained in propositions 3 and 6 is the product of country-size and market-size factors, which are, thus, the substitutes, consistently with evidence on growth, openness and country size discussed in the beginning of the article. The country-size factor is exogenous and

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two asymmetric countries with no trade costs of section 4.4. In that example $s_1 = \rho^{1/2}/(1+\rho^{1/2})$ and the stationary growth rate is $g = g(\gamma_1)(1/s_1 - 1) = g(\gamma_1)(1+\rho^{-1/2}-1) = g(\gamma_1)\rho^{-1/2} \approx g(\gamma_1)(1-(1/2)(\rho-1)) = g(\gamma_1)(3-\rho)/2$.

more important for a large economy under higher barriers to trade. The market-size factor is endogenous and defined by the ratio of the market extent to the global market share of the country. It may be decisive for growth of a small economy more open to trade.

The market-size factor of growth is preconditioned by the scale effect in the firm's activity indicated by the endogenous fixed cost. The equilibrium number of employees per firm is proportional to the net fixed cost multiplied by the ratio of the market-extent to the market share, $\gamma_i / n_{it} = \sigma\delta_i M_{it} / s_{it}$. For comparison, as was mentioned in footnote 17, the firm size in labor units in the reference case of the Krugman model is proportional to the exogenous fixed cost of production, $\gamma_i / n_{it} = \sigma f$. Contrary to the conclusion by Paul Krugman that the number and size of firms are unaffected by trade and transportation costs (1980, p. 954), the scale of production in our model depends on market pressure on the national economy and its openness to global trade.

Both determinants of firm size are apparent in the empirical evidence presented in the introduction. On the one hand, notable growth of average firm size for the case of European integration shown in figure 1 is supportive to the inference that the market extent positively affects the endogenous fixed cost and the scale of production. On the other hand, evidence on firm size and country size for the European countries depicted in figure 2 (a, b) relates to the theoretical firm-size curves $r(\gamma_i)$ drawn in figure 3. These curves demonstrate counteraction of country-size national externality and competitive pressure of trade; in particular, the U-shaped curve corresponds to moderate external effect. Points in figure 2 locate around a pronounced non-monotone curve permitting interpretation in terms of such an external effect. Note that the shape of theoretical firm-size curve $r(\gamma_i)$ does not matter for our qualitative inferences about growth rate, because the country-size factor $g(\gamma_i)$ is increasing in γ_i irrespective of the power of the country-size externality and the shape of firm-size curve $r(\gamma_i)$.

In the common market case this curve determines, via the scale effect in price setting, cross-country differentiation of the PPI indicating the technology-adjusted wage. The CPI is the same across countries in this case, but, generally, it varies because of the country-size asymmetry under trading costs. The real exchange rate thus deviates from one and defines the endogenous fixed cost and the firm size for the stationary growth path. The causality is the following. The real exchange rate is decreasing with the relative country size, and this effect is more pronounced under higher barriers to trade. The endogenous fixed cost is increasing with the real exchange rate and is, therefore, higher for a smaller economy. A firm in this economy is possessed of the larger market-size factor of growth and has stronger incentives to invest in technology improvement.

The approximate closed-form solution given in proposition 7 establishes the dependence of the stationary growth rate on the measures of relative country size and trade flow openness. Equation (47) is relevant to the growth regressions in Ades & Glaeser (1999), Alesina et al. (2000, 2004), Alcala & Ciccone (2003) and to the theoretical growth equation derived by Alesina et al. (2000, 2004). Growth in our model relates positively to country size and openness, and negatively to the product of these factors. Openness affects growth through symmetric market extent $M^{(2)}(\vartheta)$ and the term indicating the effect of country size asymmetry. The latter is positive for the small economy and negative for the large one, consistently with empirical evidence that the effect of openness on growth is diminishing in country size.

8. Concluding comments

The theorem of Adam Smith created the following dilemma: “Either the division of labor is limited by the extent of the market, and, characteristically, industries are monopolized; or industries are characteristically competitive, and the theorem is false or of little significance” (Stigler 1951, p.185). This dilemma has been resolved with the models of imperfect competition with local or temporary monopolies and external economy. Armed with these tools, the growth theory made notable advances, but an important controversial issue remained opened. The models of endogenous growth with international trade typically predict, in the absence of cross-border knowledge spillovers, concentration of R&D activity in the most advanced or largest economies. The tendency of monopolization, similar to one pointed by George Stigler, appeared to be prevailing on the global level and forcing, in the long perspective, the smaller or less advanced economies out of global technology competition. According to the endogenous growth theory, these economies can sustain competitive pressure only with the help of growth-supporting international spillovers.

We do not deny the stabilizing role of such externalities, but follow the classical tradition and make focus on the pure effects of international trade. As has been shown, the competitive pressure effect of openness can be viewed as a sufficiently strong factor of growth precluding monopolization of the global economy. The market-size factor indicates this pressure and exerts the similar effects on technology growth and global dynamics as the technology diffusion factor assumed in the models of endogenous or quasi-endogenous growth.[29]

Although the inferences of our model are consistent with cross-country observations, it is not designed to make accurate quantitative predictions of growth. Certainly, there is a room for developing this model. As an example, a firm treats equally actions of rivals from all foreign countries, advanced and less advanced. A more elaborated version of the model could allow firms to distinguish between more and less advanced competitors and to account for the distance to the global technological edge (e.g., Howitt 2000). Introduction of international knowledge spillovers could also refine the behavior of firms in the presence of free inflows of information. Depending on formal specification, the spillovers can strengthen or weaken incentives to invest in new

[29] The latter are suitable for estimating growth effects of cross-boarder knowledge diffusion. For instance, Andres Rodriguez-Clare (2007) has demonstrated, building on Eaton-Kortum's (2001) model with diffusion of ideas, that gains from international sharing of best ideas notably exceed pure gains from trade.

technology. Interaction between market-size factor and technology diffusion and the resulting growth effects can be an issue of further research.

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Appendix A: figures

Figure 1. Dynamics of average firm size

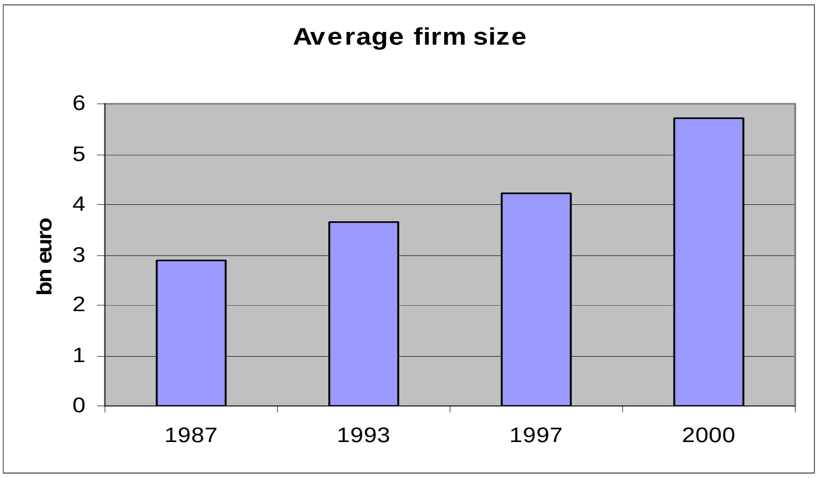
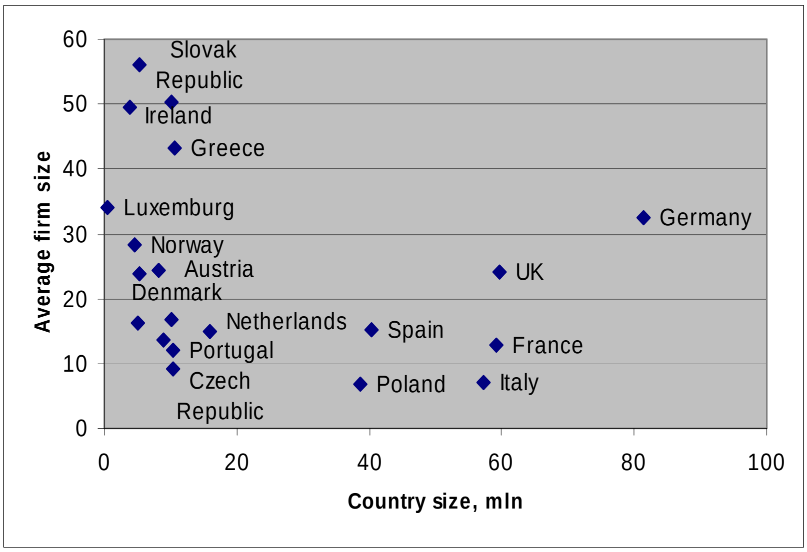
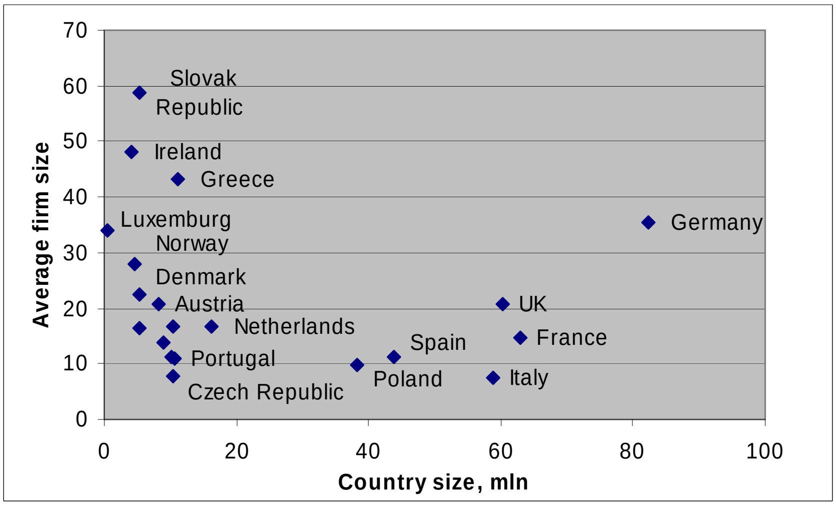


Figure 2a. Average size of manufacturing firms and country size, 2000



Source: OECD in Figures, 2003, p. 28-29. Note: Names of countries not placed on the plot are Belgium, Finland, Hungary, Sweden.

Figure 2b. Average size of manufacturing firms and country size, 2004



Source: OECD in Figures, 2006-2007, p. 20-21 (data for Greece are from OECD in Figures 2005, p. 28).
Note: Names of countries not placed on the plot are Belgium, Finland, Hungary, Sweden.

Figure 3. Dependence of firm size on country size

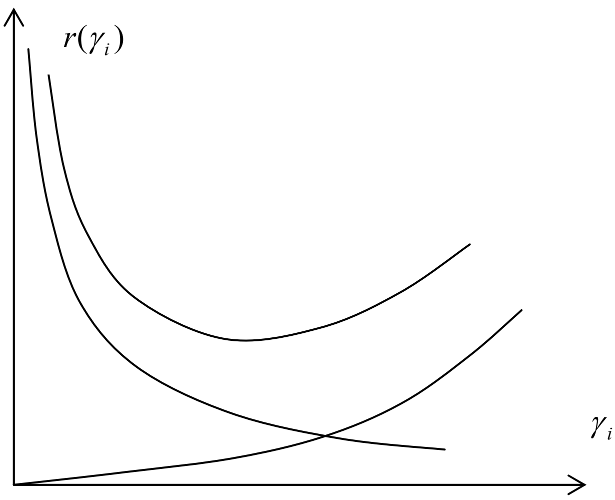


Figure 4. The domain of positive growth, $N = 2$

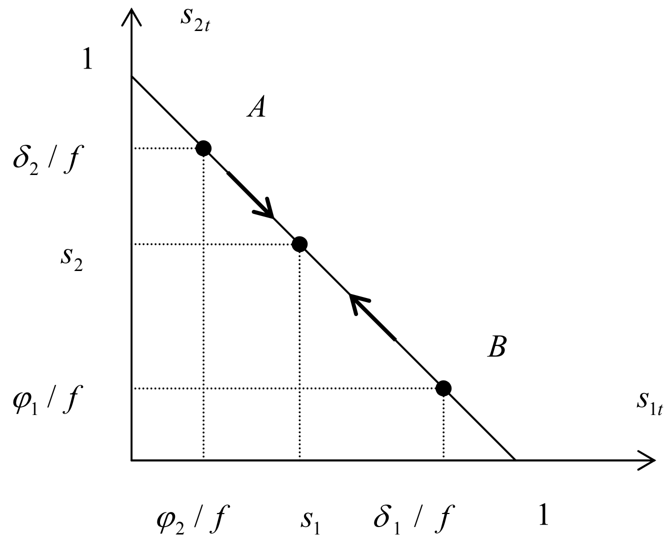


Figure 5. The zone of positive stationary growth

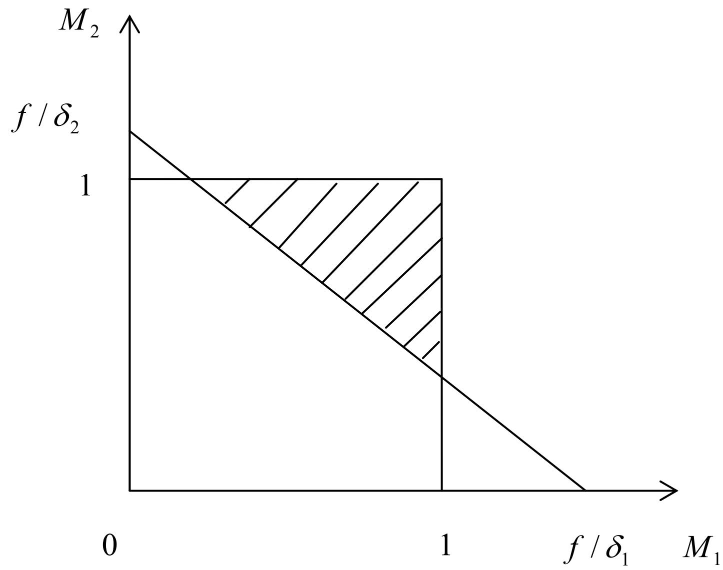


Figure 6. The steady-state equilibrium for two countries

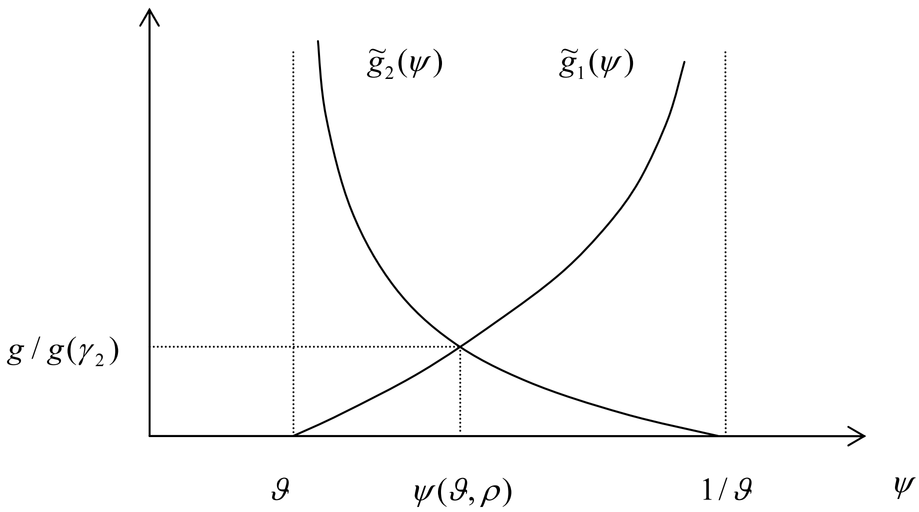
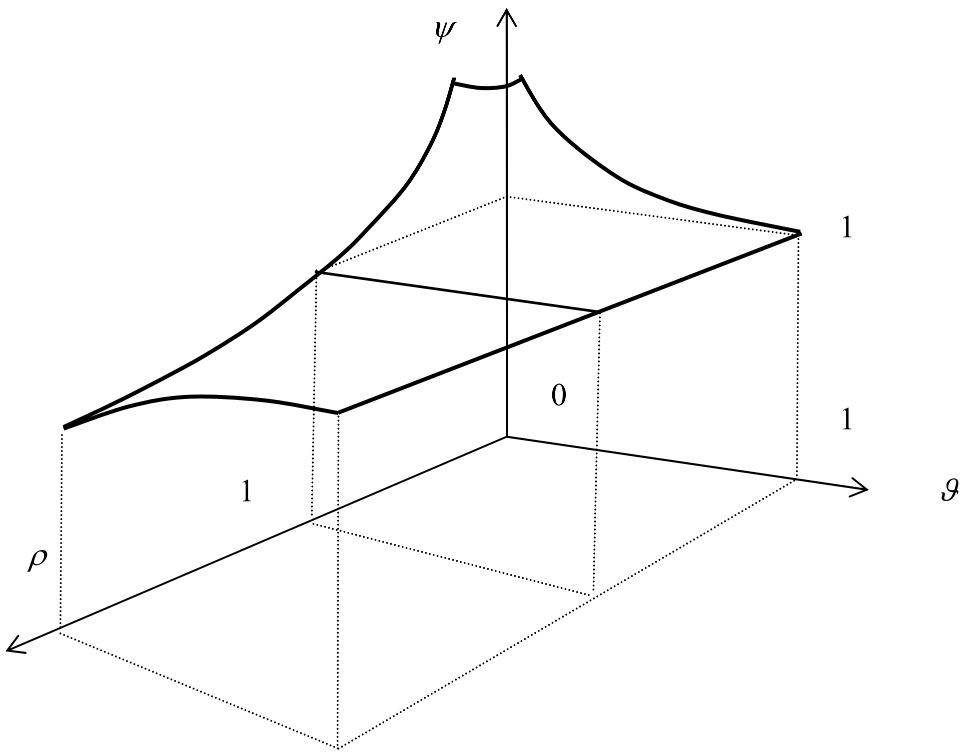


Figure 7. The steady-state real exchange rate as a function of relative country size and openness



Appendix B

Proof of proposition 1. The Lagrangian for the firm problem can be written as

$$L_i = ...\beta^t(\pi_{it} - \eta_{it}x_{it}) + \beta^{t+1}(\pi_{it+1} - \eta_{it+1}x_{it+1}) + ...$$

where $\pi_{it} = \Pi_{it} - w_{it}(x_{it} + f)$ is net profit, η_{it} is the Lagrange multiplier related to constraint $x_{it} \geq 0$. Replacing x_{it} by a_{it}, as given by (7), $x_{it} = \Delta a_{it}(\varphi_i / \bar{a}_{it-1})$, and differentiating with respect to a_{it} yields the first-order condition for period *t*:

$$\frac{\partial \pi_{it}}{\partial a_{it}} - \eta_{it}\frac{\partial x_{it}}{\partial a_{it}} + \beta\left(\frac{\partial \pi_{it+1}}{\partial a_{it}} - \eta_{it+1}\frac{\partial x_{it+1}}{\partial a_{it}}\right) = 0$$

with $\eta_{it} = 0$ for positive investment. From (7), $\frac{\partial \pi_{it}}{\partial a_{it}} = \sigma^{-1}\frac{\partial r_{it}}{\partial a_{it}} - w_{it}\frac{\partial x_{it}}{\partial a_{it}} = \sigma^{-1}\frac{\partial r_{it}}{\partial a_{it}} - \frac{w_{it}\varphi_i}{\bar{a}_{it-1}}$.

Taking into account that symmetric actions of local firms are internalized in decision to invest, we have $\frac{\partial \pi_{it+1}}{\partial a_{it}} = -w_{it+1}\frac{\partial x_{it+1}}{\partial a_{it}} = \quad w_{it+1}\varphi_i\frac{\partial(a_{it}/\bar{a}_{it})}{\partial a_{it}} = w_{it+1}\varphi_i\frac{\partial(1)}{\partial a_{it}} = 0$. The first-order condition is, hence, reduced to $\frac{\partial \pi_{it}}{\partial a_{it}} = 0$ or $\sigma^{-1}\frac{\partial r_{it}}{\partial a_{it}} = \frac{w_{it}\varphi_i}{\bar{a}_{it-1}}$.

According to (14), (15) $\frac{\partial r_{it}}{\partial a_{it}} = \frac{\sum_{k\neq i} n_{kt}h_{kit}}{a_{it}\left(n_{it} + \sum_{k\neq i} n_{kt}h_{kit}\right)^2} = \frac{s_{it}(1-s_{it})}{a_{it}n_{it}}$ because $s_{it} = n_{it}r_{it} = n_{it} / \left(n_{it} + \sum_{k\neq i} n_{kt}h_{kit}\right)$. Inserting this derivative into the first-order condition and rearranging terms yields $s_{it}(1-s_{it})/\sigma n_{it} = w_{it}x_{it} + w_{it}\varphi_i$. Combining this with the zero-profit condition $(8')$ implying that $r_{it}/\sigma = w_{it}(f + x_{it})$ or $s_{it}/\sigma n_{it} = w_{it}(f + x_{it})$ yields $s_{it}^2/\sigma n_{it} = w_{it}(f - \varphi_i)$ or $n_{it} = s_{it}^2/\sigma w_{it}\delta_i$.

Proof of proposition 2. Inserting the number of firms (16) into the market share (13) and taking into account the wage equation (17) yields: $s_{it} = n_{it}a_{it}(1+\mu)^{1-\sigma}(w_{it}/P_t)^{1-\sigma}$

$= (s_{it}^2 / \sigma w_{it}\delta_i) a_{it} (1+\mu)^{1-\sigma} (w_{it} / P_t)^{1-\sigma} = s_{it}^2 a_{it} w_{it}^{-\sigma} P_t^{\sigma-1} (1+\mu)^{1-\sigma} / \sigma\delta_i = s_{it}^{2-\sigma} a_{it} P_t^{\sigma-1} \gamma_i^{\sigma} (1+\mu)^{1-\sigma} / \sigma\delta_i$.

Hence, $s_{it}^{\sigma-1} = a_{it} P_t^{\sigma-1} \gamma_i^{\sigma} (1+\mu)^{1-\sigma} / \sigma\delta_i$ and $s_{it} = a_{it}^{\mu} \gamma_i (\gamma_i / \sigma\delta_i)^{\mu} P_t / (1+\mu) \ = a_{it}^{\mu} r_i^{-\mu} \gamma_i P_t / (1+\mu)$.

The ratio of the market shares is $s_{it} / s_{kt} = (a_{it} / r_i)^{\mu} \gamma_i / (a_{kt} / r_k)^{\mu} \gamma_k$, implying that

$$s_{it} = \frac{(a_{it} / r_i)^{\mu} \gamma_i}{\sum (a_{kt} / r_k)^{\mu} \gamma_k}.$$

Proof of proposition 4. As in the proof of proposition 1, the first-order condition for the firm problem is $\frac{\partial \pi_{it}}{\partial a_{it}} = 0$ or $\sigma^{-1} \frac{\partial r_{it}}{\partial a_{it}} = \frac{w_{it}\varphi_i}{\bar{a}_{it-1}}$. The firm's global revenue is $r_{it} = \sum_j r_{it}^j$ where according to (26), (27), the local revenue is $r_{it}^j = E^j / \left(n_{it} + \sum_{k \neq i} n_{kt} h_{kit}^j\right)$, and the relative revenue in the local market is $h_{kit}^j = (a_{kt}\tau_{ij} / a_{it}\tau_{kj})(w_{kt} / w_{it})^{1-\sigma}$. Differentiating the firm's global revenue with respect to a_{it} yields $\frac{\partial r_{it}}{\partial a_{it}} = \sum_j \frac{\partial r_{it}^j}{\partial a_{it}} = \sum_j E^j \frac{\sum_{k \neq i} n_{kt} h_{kit}^j}{a_{it}\left(n_{it} + \sum_{k \neq i} n_{kt} h_{kit}^j\right)^2} = \sum_j E^j \frac{s_{it}^j (1 - s_{it}^j)}{a_{it} n_{it}}$, since $s_{it}^j = n_{it} r_{it}^j = \ n_{it} / \left(n_{it} + \sum_{k \neq i} n_{kt} h_{kit}^j\right)$ and $1 - s_{it}^j = \sum_{k \neq i} n_{kt} h_{kit}^j / \left(n_{it} + \sum_{k \neq i} n_{kt} h_{kit}^j\right)$. The first-order condition is, hence, $\sigma^{-1} \sum_j E^j s_{it}^j (1 - s_{it}^j) / a_{it} n_{it} = w_{it} \varphi_i / \bar{a}_{it-1}$ or, after rearranging terms, $\sigma^{-1} r_{it} - \sigma^{-1} \sum_j E^j s_{it}^{j2} / n_{it} = w_{it}(x_{it} + \varphi_i)$. Combining this with the zero-profit condition (8′) implying that $\sigma^{-1} r_{it} = w_{it}(x_{it} + f)$ yields $\sigma w_{it} \delta_i = \sum_j E^j s_{it}^{j2} / n_{it}$ or $n_{it} = \sum_j E^j s_{it}^{j2} / \sigma w_{it} \delta_i$ $= s_{it}^2 / \sigma w_{it} \delta_i M_{it}$.

Proof of proposition 5. Taking into account (25), (29) and (17), the global market share is represented as $s_{it} = \sum_j E^j s_{it}^j = \sum_j E^j n_{it} b_{it}^{\sigma} (p_{it}^j / P_t^j)^{1-\sigma} = \sum_j E^j n_{it} a_{it} \left((1+\mu)\tau_{ij} w_{it}\right)^{1-\sigma} P_t^{j\sigma-1}$

$$= (1+\mu)^{1-\sigma} \left((s_{it}^2 / \sigma w_{it} \delta_i M_{it}) a_{it} w_{it}^{1-\sigma}\right) \sum_j E^j (P_t^j / \tau_{ij})^{\sigma-1}$$
$$= (1+\mu)^{1-\sigma} \left((s_{it}^{2-\sigma} / \sigma \delta_i M_{it}) a_{it} \gamma_i^{\sigma}\right) \sum_j E^j (P_t^j / \tau_{ij})^{\sigma-1}.$$

This implies $s_{it} = (1+\mu)^{-1}(a_{it}/\sigma\delta_i)^{\mu}\gamma_i^{1+\mu}M_{it}^{-\mu}F_{it} = (1+\mu)^{-1}(a_{it}/r_i)^{\mu}\gamma_i M_{it}^{-\mu}F_{it}$, where $F_{it} = \left(\sum_j E^j (P_t^j/\tau_{ij})^{\sigma-1}\right)^{\mu}$ implying (33). From (28), $(1+\mu)^{-1}r_i^{-\mu}F_{it}M_{it}^{-\mu} =$

$$(1+\mu)^{-1}r_i^{-\mu}\frac{\left(\sum_j E^j\left(P_t^{j\sigma-1}/\tau_{ij}\right)^{2(\sigma-1)}\right)^{\mu}}{\left(\sum_j E^j\left(P_t^{j\sigma-1}/\tau_{ij}\right)^{\sigma-1}\right)^{\mu}} \equiv \Phi_{it}\text{, hence } s_{it} = \Phi_{it}a_{it}^{\mu}\gamma_i .$$

The CPI is found by inserting (29) and (17) into (4) and accounting for the mark-up pricing rule and equilibrium wage $w_{kt} = \Phi_{kt}a_{kt}^{\mu}$: $P_t^j = \left(\sum_k n_{kt}b_{kt}^{\sigma}p_{kt}^{j1-\sigma}\right)^{-\mu}$

$$= (1+\mu)\left(\sum_k (s_{kt}\gamma_k/\sigma\delta_k M_{kt})(a_{kt}/\tau_{kj}w_{kt})^{\sigma-1}\right)^{-\mu} = (1+\mu)\left(\sum_k (s_{kt}/r_k M_{kt})(a_{kt}/\tau_{kj}\Phi_{kt}^{\sigma-1}a_{kt})\right)^{-\mu}$$

$= (1+\mu)\left(\sum_k (s_{kt}/r_k\tau_{kj}M_{kt}\Phi_{kt}^{\sigma-1})\right)^{-\mu}$. Since, from (28), (32) and (33), $\Phi_{kt}^{\sigma-1} = F_{kt}^{\sigma-1}M_{kt}^{-1}(1+\mu)^{1-\sigma}r_k^{-1}$, we obtain $P_t^{j1-\sigma} = \sum_k s_{kt}(F_{kt}\tau_{kj})^{1-\sigma}$.

The stationary growth path: the case of two countries

For the stationary growth path $E^j = s_j$, and, from (33), the SPI of the countries are $F_1^{\sigma-1} = s_1P^{1\sigma-1} + s_2\vartheta_{12}P^{2\sigma-1}$, $F_2^{\sigma-1} = s_1\vartheta_{21}P^{1\sigma-1} + s_2P^{2\sigma-1}$. The local market shares are calculated according to (35) as $s_i^j = s_i\vartheta_{ij}\left(P^j/F_i\right)^{\sigma-1}$:

$$s_1^1 = \frac{s_1P^{1\sigma-1}}{F_1^{\sigma-1}} = \frac{s_1(P^1/P^2)^{\sigma-1}}{s_1(P^1/P^2)^{\sigma-1} + s_2\vartheta} = \frac{s_1\psi}{s_1\psi + s_2\vartheta}$$

$$s_1^2 = \frac{s_1\vartheta_{12}P^{2\sigma-1}}{F_1^{\sigma-1}} = \frac{s_1(P^2/P^2)^{\sigma-1}\vartheta}{s_1(P^1/P^2)^{\sigma-1} + s_2\vartheta} = \frac{s_1\vartheta}{s_1\psi + s_2\vartheta}$$

$$s_2^2 = \frac{s_2P^{2\sigma-1}}{F_2^{\sigma-1}} = \frac{s_2(P^2/P^2)^{\sigma-1}}{s_1(P^1/P^2)^{\sigma-1}\vartheta + s_2} = \frac{s_2}{s_1\psi\vartheta + s_2}$$

$$s_2^1 = \frac{s_2\vartheta_{21}P^{1\sigma-1}}{F_2^{\sigma-1}} = \frac{s_2(P^1/P^2)^{\sigma-1}\vartheta}{s_1(P^1/P^2)^{\sigma-1}\vartheta + s_2} = \frac{s_2\psi\vartheta}{s_1\psi\vartheta + s_2}$$

The market-clearing equations for the local markets (40) are

$$\frac{s_1\psi}{s_1\psi + s_2\vartheta} + \frac{s_2\vartheta\psi}{s_1\vartheta\psi + s_2} = 1$$

$$\frac{s_2}{s_2 + s_1\vartheta\psi} + \frac{s_1\vartheta}{s_2\vartheta + s_1\psi} = 1$$

The global market shares satisfying these equations are $s_1 = \dfrac{1-\vartheta\psi}{1-\vartheta\psi + (\psi-\vartheta)\psi}$ and $s_2 = \dfrac{(\psi-\vartheta)\psi}{1-\vartheta\psi + (\psi-\vartheta)\psi}$.

The market extent is calculated for country 1 as (39) by taking into account (45): $M_1 = \dfrac{(s_1\psi + s_2\vartheta)^2}{s_1\psi^2 + s_2\vartheta^2} = \dfrac{((1-\vartheta\psi)\psi + (\psi-\vartheta)\psi\vartheta)^2}{(1-2\vartheta\psi+\psi^2)((1-\vartheta\psi)\psi^2 + (\psi-\vartheta)\psi\vartheta^2)}$
$= \dfrac{\psi(1-\vartheta^2)^2}{(1-2\vartheta\psi+\psi^2)((1-\vartheta\psi)\psi + (\psi-\vartheta)\vartheta^2)}$. Similarly, for country 2 $M_2 = \dfrac{(s_2 + s_1\psi\vartheta)^2}{s_2 + s_1\psi^2\vartheta^2} =$

$= \dfrac{((\psi-\vartheta)\psi + (1-\vartheta\psi)\psi\vartheta)^2}{(1-2\vartheta\psi+\psi^2)((\psi-\vartheta)\psi + (1-\vartheta\psi)\psi^2\vartheta^2)} = \dfrac{\psi^2(1-\vartheta^2)^2}{(1-2\vartheta\psi+\psi^2)((1-\vartheta\psi^{-1}) + (1-\vartheta\psi)\vartheta^2)}$.

These coincide with the market extent for symmetric countries (44): if $\psi = 1$, then

$M_1 = M_2 = M^{(2)}(\vartheta)$.

The ratio of market extent to market share for country 1 is

$\dfrac{M_1}{s_1} = \dfrac{\psi(1-\vartheta^2)^2}{(1-\vartheta\psi)((1-\vartheta\psi)\psi + (\psi-\vartheta)\vartheta^2)} = \dfrac{(1-\vartheta^2)^2}{(1-\vartheta\psi)^2 + (1-\vartheta\psi)(1-\vartheta/\psi)\vartheta^2}$, and, consequently,

from (38), $g = g(\gamma_1)(M_1/s_1 - 1) = g(\gamma_1)\left(\dfrac{(1-\vartheta^2)^2}{H(\psi,\vartheta)} - 1\right)$, where

$H(\psi,\vartheta) = (1-\vartheta\psi)^2 + (1-\vartheta\psi)(1-\vartheta/\psi)\vartheta^2$. Similarly, $\dfrac{M_2}{s_2} = \dfrac{\psi^2(1-\vartheta^2)^2}{(\psi-\vartheta)\psi((1-\vartheta/\psi) + (1-\vartheta\psi)\vartheta^2)}$
$= \dfrac{(1-\vartheta^2)^2}{(1-\vartheta/\psi^2 + (1-\vartheta\psi)(1-\vartheta/\psi)\vartheta^2}$ and $g = g(\gamma_2)(M_2/s_2 - 1) = g(\gamma_2)\left(\dfrac{(1-\vartheta^2)^2}{H(1/\psi,\vartheta)} - 1\right)$, where

$H(1/\psi,\vartheta) = (1-\vartheta/\psi)^2 + (1-\vartheta\psi)(1-\vartheta/\psi)\vartheta^2$.

Proof of proposition 7. For ρ close to 1 the growth rate for country 1 is

$$g = g(\gamma_1)\left(\frac{(1-\vartheta^2)^2}{H(\psi,\vartheta)} - 1\right) \approx g(\gamma_1)\left(\frac{(1-\vartheta^2)^2}{H(1,\vartheta)} - \frac{(1-\vartheta^2)^2}{H(1,\vartheta)^2} H'_\psi(1,\vartheta)\psi'_\rho(\vartheta,1)(\rho-1) - 1\right) \qquad \text{(A1)}$$

Since $H(1,\vartheta) = (1-\vartheta)^2(1+\vartheta^2)$, the first term in the brackets of (A1) is $\dfrac{(1-\vartheta)^2(1+\vartheta)^2}{(1-\vartheta)^2(1+\vartheta^2)}$

$= \dfrac{(1+\vartheta)^2}{1+\vartheta^2} = 2M^{(2)}(\vartheta)$.

Consider the second term. We have that $H'_\psi(\psi,\vartheta) = -2\vartheta(1-\vartheta\psi) - \vartheta^3 + \vartheta^3/\psi$ and, hence, $H'_\psi(1,\vartheta) = -2\vartheta(1-\vartheta)$. To calculate $\psi'_\rho(\vartheta,1)$ apply the implicit function theorem to (46) represented as $G \equiv \rho\left(\dfrac{(1-\vartheta^2)^2}{H(\psi,\vartheta)} - 1\right) - \dfrac{(1-\vartheta^2)^2}{H(1/\psi,\vartheta)} + 1 = 0$:

$$\frac{\partial G}{\partial \rho} = \frac{(1-\vartheta^2)^2}{H(\psi,\vartheta)} - 1\Bigg|_{\psi=1} = \frac{(1-\vartheta^2)^2}{(1-\vartheta)^2(1+\vartheta^2)} - 1 = \frac{(1+\vartheta)^2}{1+\vartheta^2} - 1 = \frac{2\vartheta}{1+\vartheta^2};$$

$$\frac{\partial G}{\partial \psi} = -\rho\frac{(1-\vartheta^2)^2}{H(\psi,\vartheta)^2}H'_\psi - \frac{(1-\vartheta^2)^2}{H(1/\psi,\vartheta)^2\psi^2}H'_{1/\psi}\Bigg|_{\substack{\rho=1,\\ \psi=1}} = -\frac{(1-\vartheta^2)^2(H'_\psi + H'_{1/\psi})}{\left((1-\vartheta)^2(1+\vartheta^2)\right)^2}.$$

Since $H'_{1/\psi}(1,\vartheta) = H'_\psi(1,\vartheta) = -2(1-\vartheta)\vartheta$, $\dfrac{\partial G}{\partial \psi} = \dfrac{4(1-\vartheta^2)^2(1-\vartheta)\vartheta}{\left((1-\vartheta)^2(1+\vartheta^2)\right)^2}$ and

$$\psi'_\rho(\vartheta,1) = -\frac{\partial G/\partial\rho}{\partial G/\partial\psi}\Bigg|_{\rho=1} = -\frac{2\vartheta}{(1+\vartheta^2)}\frac{(1-\vartheta)^4(1+\vartheta^2)^2}{4(1-\vartheta^2)^2(1-\vartheta)\vartheta} = -\frac{(1-\vartheta)(1+\vartheta^2)}{2(1+\vartheta)^2}.$$

Consequently, the second term in the brackets of (A1) is

$$\frac{(1-\vartheta^2)^2}{H(1,\vartheta)^2}H'_\psi(1,\vartheta)\psi'_\rho(1,\vartheta)(\rho-1) = \frac{(1-\vartheta)^2(1+\vartheta)^2}{(1-\vartheta)^4(1+\vartheta^2)^2}2\vartheta(1-\vartheta)\frac{(1-\vartheta)(1+\vartheta^2)}{2(1+\vartheta)^2}(\rho-1)$$

$= \dfrac{\vartheta}{1+\vartheta^2}(\rho-1)$. As a result, $g = g(\gamma_1)\left(2M^{(2)}(\vartheta) - \dfrac{\vartheta(\rho-1)}{1+\vartheta^2} - 1\right)$.