SHARE AUCTION*

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A new share auction design (large equity stakes of a company are sold simultaneously) is introduced as an alternative for selling the company as a whole. Besides the increase in number of bidders because of the budget constraint weakening, the auctioneer has the possibility to increase the revenue due to additional competition for the control.

INTRODUCTION

The distribution of the shares through auctions attracts great attention in Initial Public Offering literature (Ausubel, 2002; Armstrong, 2000). The most important thing about a selling method choice is to secure liquidity of following trades. There's no need to sell large stakes. It practice, one usually chooses the design that provides the widest spreading between bidders. As a rule, such auctions attract portfolio and private investors.

In our model the emphasis is on distribution of shares among the strategic investors, so the relatively small number of parts is implied, with a total share close to 100%.

The essential moment in such share auction is a possibility to gain control over the company. One of the classic papers on the vote value (Grossman, 1980) shows the takeover decision depends on operating control benefits. Grossman argues that if the company's charter allows a large stakeholder to increase its profit at the expense of the minority stakeholders, then it has the possibility to sell its stake above the market price. In our paper we show that an existing uncertainty of the vote value could be transformed into additional auctioneer's profit by means of using the share auction.

In general, share auction is a multiple object auction, where objects are purely neither complementary nor substitute. Let's assume that the owner is auctioning 5 identical stakes, each 20% shares. Then the bidder, who acquires two stakes, would have the blocking stake and could secure from unfavorable decisions; the bidder who buys three stakes, would control the company. So, the more stakes are acquired the more voting rights is possessed by buyer. Any additional unit increases the value of previous ones until full control is gained. Acquiring consequent stakes (fourth or fifth) decreases the value of each stake.

Palfrey (1983) shows that in case of independent objects the optimal auction's design depends on the number of bidders. If there are only few of them then the expected auctioneer's profit is larger when the objects are sold as a single bundle. As the number of participants is increasing, at some point it becomes better for the seller to sell objects separately.

Our article consider the case when two stakes are sold (50% plus 1 share and 50% minus 1 share). Because they are not independent we show there exist examples where even with few bidders the share auction is more profitable.

1. MODEL DESCRIPTION

Two parameters are used to describe stake value. Firstly, let's assume that each bidder has its own whole company value estimation x_i, which is equal to the net present value of cash flows. x_i is a realization of a random variable $\hat{x}_i$ with equal distribution function $F_x(x),\ x \in [0,+\infty)$.

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Secondly, one needs a parameter that shows the control benefits. According to the present value of the future dividends, prices of the stakes should be equal (if a marginal share is a negligible fraction). Still the non-symmetrical voting rights lead to the existence of control premium $\Pr_i$ or, respectively, discount D_i.

Premium and discount values depend on three factors:

- Whole firm value estimation, as a scaling measure;
- Investment climate;
- Corporate culture.

The second parameter provides us with a range of possible premium (discount) values and concerned with them risks. Obtaining such information and proceeding from her own corporate policy, a bidder chooses the appropriate premium value, and the discount value comes through knowledge about the other bidders. So, in general, $D_i \neq \Pr_i$. In symmetric case, $D_i = \Pr_i$.

So we assume that $\Pr_i = D_i = d_i \frac{x_i}{2}$ where d_i is the independent of x_i, being the realization of the random variable $\hat{d}_i$ with common probability distribution function $F_d(d),\ d \in [0,1]$. The closer d to 1, the more predatory is the large stakeholder's policy.

After defining basic model's parameters, let's proceed to the basic assumptions:

1. Private values. Each bidder has her own value estimation (x,d) and has no knowledge about other bidder's values.
2. Quasi-linear utility function. The winner of the one of the stakes gets the profit of $v-c$, where v is for her private value, c is for costs.
3. Zero start value. Each auction starts with zero price.
4. The number of bidders is well-known and is equal to n.

The auction form could be quite different. Parallel auction is possible, when different stakes have their own auctions, or combinatorial auction, with both stakes bids taken simultaneously. Here we observe the parallel auction from the auctioneer's view and its dependence on d distribution.

2. PARALLEL AUCTIONS

Each participant's bid is 2-dimentional vector $\vec{b} = (b_1, b_2)$. Since the auctions are independent, we may optimize them independently, and total bidder's profit will be $\hat{\pi} = \hat{\pi}_1 + \hat{\pi}_2$, the sum of profits of each auction.

Auctions may have any of the standard form: sealed-bid auction of the first (second) price, English or Dutch auction. We show that, in accordance with our assumptions the following proposition is correct.

Proposition

A) In a sealed-bid second-price auction the optimal bid $b_i^{II}(x,d)$ is equal to the stake value estimation

$$b_i^{II}(x,d) = \frac{x}{2}\left[1+(-1)^{i+1}d\right]$$

B) In a sealed-bid first-price auction the optimal strategy $b_i^{I}(x,d)$ *is*

$$b_i^{I}(x,d) = \frac{x}{2}\left[1+(-1)^{i+1}d\right] - \frac{\displaystyle\int_0^{\frac{x}{2}\left[1+(-1)^{i+1}d\right]} dy \left[\iint_{\frac{u}{2}\left[1+(-1)^{i+1}v\right]<y} f_x(u) f_d(v)\,du\,dv\right]^{n-1}}{\left[\displaystyle\iint_{\frac{u}{2}\left[1+(-1)^{i+1}v\right]<\frac{x}{2}\left[1+(-1)^{i+1}d\right]} f_x(u) f_d(v)\,du\,dv\right]^{n-1}},$$

where i=1 is for control stake auction, i=2 is for blocking stake auction .

The proof is straightforward if the new variables are introduced, $y_i = \frac{x}{2}\left[1+(-1)^{i+1}d\right]$. After that the derivation could be made in course of common auctions theory. The direct proof could be find in Kuchaev (2006).

3. AUCTIONEER'S REVENUE

As in the common auction theory, the auctioneer's revenue in the second-price auction is the expectation of the second highest value, $R_i^{II} = B_{2,i}^{(n)} = E[Y_{2,i}^{(n)}]$, and in the first-price auction – the function of optimal bid from the winner's value $R_i^{I} = B_{1,i}^{(n)} = b_i^I\left(Y_{1,i}^{(n)}\right) = E[Y_{2,i}^{(n)}]$. So the auctioneer's revenue is the same in both cases and is equal to $R = E[Y_{2,1}^{(n)} + Y_{2,2}^{(n)}] = \int_0^{+\infty} y\left(f_{Y_{2,1}^{(n)}}(y) + f_{Y_{2,2}^{(n)}}(y)\right)dy$, where

$f_{Y_{2,i}^{(n)}}(y) = n(n-1)(1 - F_{y,i}(y))F_{y,i}^{n-2}(y)f_{y,i}(y)$, and $F_{y,i}(y) = \iint\limits_{\frac{u}{2}\left[1+(-1)^{i+1}v\right]<y} f_x(u)f_d(v)dudv$

In general, the revenue couldn't be computed analytically, so we consider the share auction in specific cases.

4. SPECIAL CASES

CASE A

Lets $\hat{d}$ has the only realization, d_0. Then the estimated auctioneer's revenue will be $R_0^{(n)} = \int_0^{+\infty} yn(n-1)(1 - F_x(y))F_x^{n-2}(y)f_x(y)dy$. $R_0^{(n)}$ doesn't depend on d_0 and is the same as on common auctions where company is sold as a whole (Krishna, 2002).

CASE B

Lets $\hat{d}$ has bimodal distribution with realizations d_1, d_2 with equal probability. Taking average and deviation $m_d = \frac{d_1 + d_2}{2}, \sigma_d = \frac{d_2 - d_1}{2}$, we obtain for small σ_d ($\sigma_d << 1 - m_d$) and quite smooth $F_x(.)$

$$R \approx R_0^{(n)} + \frac{n(n-1)(n-2)}{2}\frac{\sigma_d^2}{1 - m_d^2}\int_0^{+\infty} z^2 f_x^2(z)\left(1 - \frac{n-1}{n-2}F_x(z)\right)F_x^{n-3}(z)dz.$$

The auctioneer's revenue may rise or fall, dependent of the number of bidders and distribution function $F_x(.)$. If there only two bidders, the second term of the equation is always negative.

CASE C

Here $\hat{d}$ again takes one of two values d_1, d_2, and x is uniformly distributed on $[x_1, x_2]$. In this case the auctioneer's revenue could be obtained analytically. Here we present only numerical results in terms of the share auction profitability, $r^{(n)} = \frac{R^{(n)} - R_0^{(n)}}{R_0^{(n)}}$.

Pictures 1 and 2 present the results for the moderate premium of $m_d = 10\%$ and for different number of bidders. For small $\frac{x_1}{2\Delta}$ and small $\frac{\sigma_d}{m_d}$, it's profitable to sell company as a whole, but when parameters are rising, the share auction becomes better. In contrast to the Palfrey the share auction may be profitable even with a small number of bidders. With n growth the negative $r^{(n)}$ value area converges, and the profitability of the share auction rises. The crucial role is for x distribution, hardly determined in practice. On the other side, the d distribution may be managed by the auctioneer, who can improve auction results by expanding possible deviation σ_d, which is in line with Grossman findings. The excessive expand, however, could lead to the violation of equality $D_i = \Pr_i$.

5. SOME CASES OF THREE STAKES

For simplicity we consider here Vickrey-Clarke-Grove (VCG) auction. It is efficient, incentive compatible and individually rational mechanism. Also VCG auction is most profitable for auctioneer among all efficient mechanisms. Detailed description one can find in Krishna (2002).

The auctioneer's revenue also depends on the way of dividing the equity. One way is to divide it equally in 3 parts. In this case each part would contain 1/3 of equity, and the number of combinations would be equal to 3.

TABLE. *Set of all possible combinations*

Package	***Share in equity***	***The value of package***
A	1/3	$\frac{x}{3}-\frac{xd}{2}$
B	2/3	$\frac{2x}{3}+\frac{xd}{2}$
C	1	x

As the packages are additive, the VCG auction becomes identical to parallel auction described in Section 2.

In more complicated case the parts are not identical. Let's consider next division: (*A*) 50%+1 share, (*B*) 25%+1 share and (*C*) 25%-1share. In this case we have 7 possible packages and in accordance with Section 1 they would have following valuations:

TABLE. *Set of all possible combinations*

Package	***Share in equity***	***The value of package***
A	50%+1	$\frac{x}{2}+\frac{xd}{2}$
B	25%+1	$\frac{x}{4}-\frac{xd}{2}$
C	25%-2	$\frac{x}{4}-\gamma\frac{xd}{2},\ \gamma>1$
$A\cup B$	75%+2	$\frac{3x}{4}+\gamma\frac{xd}{2}$
$A\cup C$	75%-1	$\frac{3x}{4}+\frac{xd}{2}$
$B\cup C$	50%-1	$\frac{x}{2}-\frac{xd}{2}$
$A\cup B\cup C$	100%	x

The γ is introduced here to make difference between *B* and *C*. It reveals the fact that the latter package helds less voting rights.

If there are only two bidders, it is always better to sell the company's equity as a whole (Krishna, 2002). With three buyers it can be shown on example that separate selling may be more profitable. Suppose we have following realization

TABLE. *An example data*

	1	***2***	***3***
x	60	65	70
d	0.2	0.3	0.4
γ	1.1	1.05	1.05
A	36.0	42.3	49.0
B	9.0	6.5	3.5
C	**8.4**	6.0	2.8
$A\cup B$	51.6	59.0	**67.2**
$A\cup C$	51.0	58.5	66.5
$B\cup C$	24.0	22.8	21.0
$A\cup B\cup C$	60.0	65.0	**70.0**

Without bundling, the efficient allocation is to give C to buyer 1 and $A\cup B$ to buyer 3. The total revenue in the VCG mechanism would be 65.1. If the equity was sold as a whole, then it would go to buyer 3 and he would pay 65.0.

6. SHARE AUCTION APPLICATION

Mutual bidding could be a circumstantial evidence of the share selling expediency. Numerical estimation shows that for distribution functions mentioned, the fall in revenue is not substantial, being about several percent. On the other hand, an appropriate choice of the number of parts could lead to independent bidding, with implied rise of the bidders number and, correspondingly, the auctioneer's expected revenue.

Shelf rights selling experience shows that oil companies were often involved to alliances and joint bidding. Hendricks (2003) introduces data on 12 largest oil companies activity during 1954-1970 common auctions.

TABLE. *Wildcat bidding by the 12 firms and consortcia, 1954-1970*

	Bids	
	sole	***joint***
Arco/Getty/Cities/Cont.	437	114
Standard Oil of California	408	76
Standard Oil of Indiana	132	276
Shell Oil	444	3
Gulf Oil	201	81
Exxon	325	42
Texaco	114	178
Mobil	48	163
Union Oil of California	95	201
Phillips	98	65
Sun Oil	241	93
Forest	195	0

The table shows off clearly that the rate of the joint bids was relatively high.

CONCLUSION

An alternative method of a company selling through share auction was introduced. Particularly, the paper discusses the case when two stakes of 50%+1 share and 50%-1 share are sold through auction. Such division has several advantages, specifically making bidder's budget constraint less strict. From the one hand, the assymmetry of the stakes retain the possibility for the bidders to compete for the control over the firm, and from the other hand, it raises the uncertainty concerning stakes valuation, affecting the auctioneer's revenue.

It is shown that share auction profitability strongly depends on distribution function $F_x(x)$, hardly known *a priori*.

Numerical calculations demonstrate that share auction profitability increases along with control valuation uncertainty (parameter manageable by the auctioneer) and with number of bidders.

In practice, share auction could be used in privatizing big companies, when the sum of transaction is high, and in oil&gas rights selling, when the risks implied are high, and firms are trying to diversify it.

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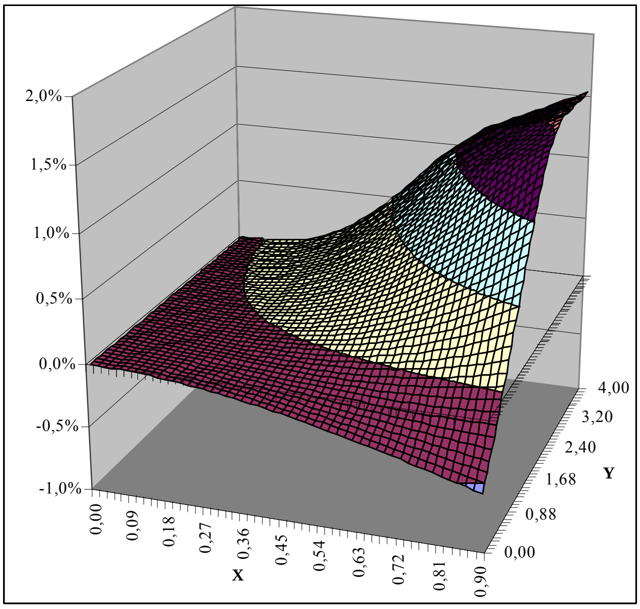
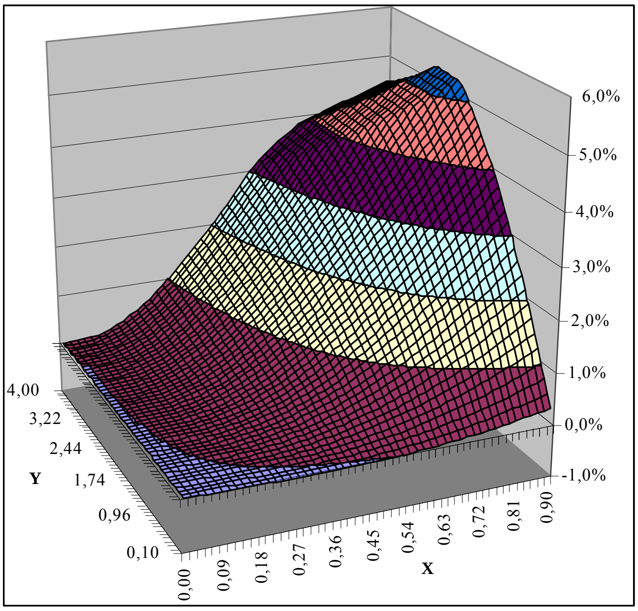
Picture 1. Profitability of share auction ($n = 4$)



X axis is for $\frac{\sigma_d}{m_d}$, Y axis is $\frac{x_1}{2\Delta}$.

Picture 2. Profitability of share auction ($n = 10$)



X axis is for $\frac{\sigma_d}{m_d}$, Y axis is $\frac{x_1}{2\Delta}$.