CLUB CONVERGENCE AND THE CAPITAL MARKET CLUB

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Abstract. This paper aims to explain club convergence by appealing to cross-country diversity in structural parameters related to quality of institutions and by viewing the international capital market as an important device of club formation. It is shown that club convergence and the limited access to the global capital market are closely interrelated features of the global economy. The lack of capital flows from rich to poor countries and the emergence of a "capital market club" are explained by the heterogeneity of countries in terms of growth-supporting institutions and by taking into account a fairly weak requirement that the global capital market forbids exponential running of net external debt by any country. Less advanced economies are unable to adjust to the global economy dynamics in terms of leisure and capital position, but may be able to grow at various rates in autarky. The paper thus provides an explanation for the stylized fact that leading countries have had nearly equal long-run growth rates, and the growth rates have varied enormously across developing economies.

Introduction

A stylized fact of global economic growth is that per capita incomes and growth rates converge in absolute terms among industrial economies but diverge for the whole world. As L. Pritchett points it out, "a set of leading countries has had nearly equal growth rates over long haul", and at the same time "the growth rates over the 1960-92 period have varied enormously across developing countries" (Pritchett (2003), p. 127).[1] The related theoretical concept is club convergence defined as that per capita incomes of countries identical in structural characteristics (preferences, technologies, rates of population growth, government policies, etc.), "converge to one another in the long run provided that their initial conditions are similar as well" (Galor 1996, p. 1056). This definition can be extended to club convergence in growth rates if per capita incomes are measured in units of effective labor or total factor productivity.

In formal models of growth, club convergence is usually a consequence of the multiplicity of long-run steady states and a threshold property establishing convergence of incomes for structurally identical economies with initial conditions in the domain of attraction to the same steady state (Galor 1996). Successes or failures of nations are thereby explained by "history" rather than structural characteristics.[2] Countries with initial level of capital and output above or below the threshold tend to high or low-equilibrium steady state, respectively. Consequently, countries with close initial level of capital on different sides of a threshold will diverge from one another.

There are indeed many cases when economies with similar initial levels of capital have been developing in quite various ways. The most striking instances are West and East

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[1] There are papers surveying empirical evidence on divergence of growth rates, for instance, Pritchett (1997), Durlauf & Quah (1999) and Easterly & Levine (2001).

Germany, Finland and the Baltic States, South and North Korea, South Korea and the Philippines. But in fact, these and other countries have diverged due to the adoption of different institutions rather than slight distances in initial volumes of capital. The emphasis on initial conditions rather than structural diversity of economies is inconsistent with the generally acknowledged fact that institutions matter for growth.

This paper aims to explain the property of club convergence by appealing to cross-country diversity in structural parameters related to quality of institutions. The problem is that rates of long run endogenous growth depend on these parameters and generally do not converge in absolute terms if economies are heterogeneous. Convergence clubs can then arise only if countries within certain groups are structurally identical, or there is a mechanism of interconnections between structurally heterogeneous economies that ensures equalization of long-run growth rates for a group of countries.

There are very few fully specified models of the global economy growth with international linkages like international trade or technology transfers. For instance, J. Ventura (1997) has shown that international trade in intermediate goods resulting in factor price equalization allows an alternative explanation of conditional convergence finding in terms of cross-country variation in growth rates. D. Acemoglu and J. Ventura (2002) infer that in the absence of diminishing returns in production, international trade leads to convergence of growth rates and a stable world income distribution. In their model, faster growth of a country worsens its terms of trade and reduces the rate of return thus slowing further capital accumulation.[3]

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[2] This is a key feature of the underdevelopment trap theory, e.g. in Murphy et. al. (1989), Azariadis & Drazen (1990), Krugman (1991), Azariadis (1996).

[3] The possibility of club convergence due to cross-country technology transfers has been demonstrated by P. Howitt (2000) and P. Howitt & D. Mayer-Foulkes (2002) for the models of global productivity growth. In Howitt (2000), this property is a consequence of the Kuhn-Tucker generalization of research arbitrage

Our paper views the international capital market as an important device of club convergence in growth rates. Empirical evidence suggests that foreign investment flows spread merely between industrial countries, whereas less developed nations face a lack of capital in spite of potentially higher returns. Importantly, foreign investment provide cross-country technology and knowledge transfers, but backward nations that extremely need these transfers lack investment inflows just because of their backwardness. Club convergence in growth rates (equalization for leading countries and diversity for lagging ones) and limited access to the global capital market are treated here as closely interrelated features of the global economy. This link is clarified by appealing to heterogeneity of countries in terms of growth-generating institutions. The paper thus focuses on substantially endogenous obstacles to economic integration stemming from the heterogeneity of the world economy. To simplify this analysis, we abstract from essentially exogenous factors constraining international capital flows like political risks and monopoly control over trade emphasized, for instance, by Lucas (1990). The lack of capital flows to economies with *relatively* poor institutions and the emergence of a "capital market club" of economies with advanced institutions are explained by taking into account a fairly weak requirement that the global capital market forbids exponential accumulation of net external debt indefinitely by any country.

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equation forbidding negative intensity of research. Club convergence is a possible equilibrium outcome in this model but it does not necessarily stem from the structural diversity of the world economy. In Howitt & D. Mayer-Foulkes (2002), this property relates to the choice between R&D or technological implementation through the adoption of R&D-supporting institutions by less advanced countries. There are three convergence clubs, characterized by R&D, implementation and stagnation. This is a consequence of multiple equilibria and the following threshold property: the country is trapped into stagnation if it fails to use effectively technological transfers in finite time and its productivity falls below a threshold.

We utilize a modified Lucas-Uzawa two-sector endogenous growth model (Lucas (1988), Uzawa (1965)) with leisure preference.[4] It is extended to the global economy where national economies differ in marginal productivities of growth-generating sectors of knowledge accumulation, capital is a mobile factor of production, and labor is an immobile factor. This extension is similar to the global economy model with free capital movement as outlined by Lucas (1993) where countries differ only in initial conditions.

It is assumed that international interest rate equalization occurs at any instant of time as a result of capital reallocation across countries and leads to equalization of labour price. Due to immobility of labour force, households in each country – "member" of the club – have to adjust consumption and the allocation of time between production, knowledge accumulation and leisure. As a result, the equilibrium ratio of consumption to qualified leisure (measured in units of human capital) is proportional to the wage rate, which is the same across countries. Consumption grows at the same rate and, hence, qualified leisure also grows at the same rate. This is possible because of instantaneous adjustment of time allocation, implying that households in countries with inferior growth-generating sectors are forced to spend less time on leisure than households in advanced countries.

Such a mechanism of adjustment implies a close cross-country interdependence of household behavior in terms of time allocation. Leisure preference proves to be an

[4] This modification was suggested by Rebelo (1991), and Jones et al. (1993), then developed by Benhabib & Perli (1994), Ortiguera & Santos (1997), Ladron-de-Guevara et al. (1999). The crucial feature of this model is that the level of human capital does not change the marginal utility of leisure. By entering leisure into the model this way, Rebelo (1991) and Jones et al. (1993) focus on the issues of long-term effects of taxation on growth, and Benhabib & Perli (1994) consider indeterminacy of solutions for a wide range of plausible model parameters. Ortiguera, S. & M. Santos (1997) examine the speed of convergence and Ladron-de-Guevara et al. (1999) demonstrate the existence of equilibrium path and reveal three balanced growth paths such that two are interior and one is a corner solution.

important property of the global economy model of growth stipulating realistic transition dynamics with a finite speed of convergence to the steady state. If households are indifferent to leisure, as in the basic Lucas (1988) model, then interest rate equalization implies a counterfactual outcome when all capital and production concentrates instantaneously in the most advanced countries.

The equalization of growth rates occurs in the long run because economies accumulate human capital at different speed. Households in advanced countries spend more time on leisure and less time on other activities as compared to autarky. Consequently, growth in these countries slows down. At the same time, households in lagged countries are forced by the global capital market to intensify efforts in production and accumulation of knowledge thus fostering economic growth.

The aforementioned no-exponential debt condition means that net foreign assets of all countries trading in the capital market are non-negative in the steady state. This is fulfilled if the marginal productivity of the sector of knowledge does not vary much across countries. Such a provision is quite restrictive if households are patient and it means that economies with less advanced growth-generating sectors do not participate in the world capital market. Low productivity of these sectors is interpreted in the sense of poor quality of growth-supporting institutions. As shown below, less advanced countries are unable to adjust to the global capital market in terms of leisure and capital position (the ratio of household assets to production capital), but may be able to grow endogenously in autarky. If so, long-run growth rates vary among countries beyond the club and are equalized within it. We demonstrate that heterogeneity of economies in terms of growth-supporting institutions rather than production technology or household preferences constitutes a crucial barrier to worldwide economic integration via the capital market.

Analysis of transitional dynamics for the club economy demonstrates that if countries have the same quality of institutions, the allocation of capital and consumption growth are defined by an initial distribution of the knowledge-to-asset ratio across these countries. Leading economies, where this ratio is relatively high, increase consumption rates above the average level along the transition path. Consumption and investment demand in these countries serves a driving force for transition growth in other economies – members of the club.

In the absence of international knowledge transfers, an economy with inferior institutions can become a member of the capital market club in a very specific pattern if it transforms to a *rentier* economy. In this case all national assets are invested abroad, all initial production capital evaporates from the country, and human capital is not utilized in production. The capital position is infinity, households devote all time to leisure, and their per capita assets grow at the worldwide rate. As shown below, such a transformation occurs in finite time.

Opportunities of less advanced countries widen due to knowledge transfers induced by foreign investment in production capital. These transfers are introduced for a small economy open to the world capital market similarly to the assumptions about spillover effects of direct foreign investment suggested earlier by Findlay (1978) and Wang (1990). These authors did not use dynamic optimization and applied models of growth with exogenous technical progress. They demonstrated that knowledge-transferring foreign investment remove a growth gap between advanced and backward economies. Our conclusions are similar though somewhat ambiguous: foreign investment can mitigate but not remove barriers to integration faced by a backward economy. If households are quite patient, this effect of investment-induced knowledge transfers is negligible.

The rest of the paper consists of five sections, a supplement and a conclusion. Section 2 examines the closed economy model, sections 3 through 5 extend it to the global economy, and supplement deals with a small open economy. Proofs of propositions and statements are collected in the appendix.

2. The autarky economy.

The economy is populated with representative agents endowed with two production factors, physical capital and labor. These agents make consumption-investment decisions that maximize discounted utility on an infinite time horizon. The number of workers equals the number of population and assumed constant.

The individual decision problem is

$$\max_{c,l,e,u,k,h} \int_0^\infty e^{-\delta t}(\ln c + \theta \ln l)dt, \tag{1}$$

$$\dot{k} = y - c, \tag{2}$$

$$\dot{h} = \psi eh, \tag{3}$$

$$u + e + l = 1, \tag{4}$$

$$e \geq 0. \tag{5}$$

Instantaneous utility is a log-additive function of consumption c and leisure l. Individual preferences are described with elasticity of leisure θ, and individual discount rate δ. Production technology is Cobb-Douglas with Harrod-neutral technical progress: $y = k^{\alpha}(uh)^{1-\alpha}$, where y is per capita output, k is physical capital per worker, h is the number of efficiency units or human capital (knowledge) of the worker, α is the share of capital in output, and u is the intensity of labor inputs in production.

Equation (2) is the budget constraint, and (3) relates to human capital accumulation. Physical capital does not depreciate. The term ψeh in (3) is a homogenous production function with human capital as a sole factor of knowledge accumulation and marginal productivity ψ. Equation (4) is a balance of time divided between leisure and production of goods and knowledge. Constraint (5) restricts effort to produce knowledge from being negative.[5]

Equilibrium dynamic in the endogenous growth regime is represented by three key ratios: consumption rate $x = c/k$, interest rate $r = \partial y/\partial k$, and leisure l.

Proposition 1. The trajectory of growth satisfies the system

$$\dot{x}/x = x - \beta r - \delta, \tag{6}$$

$$\dot{r}/r = \beta(\psi(1-l) - r), \tag{7}$$

$$\dot{l}/l = \psi u - \delta, \tag{8}$$

where $\beta = (1-\alpha)/\alpha$ *, and*

$$u = \beta r l/\theta x\,. \tag{9}$$ [6]

Output, consumption, and both types of capital are growing exponentially along balanced growth path with the same constant rate of growth while proportions of time allocation are held constant. The steady state equations for (4), (6)-(9) are

$$x = \beta r + \delta, \tag{10}$$

$$r = \delta + \psi e, \tag{11}$$

$$u = \delta/\psi, \tag{12}$$

$$l = \theta(1 + \delta/\beta r)\delta/\psi. \tag{13}$$ [7]

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[5] Production effort and leisure are always positive in equilibrium, and we ignore the corresponding constraints.

[6] This proposition is a consequence of the first-order conditions derived for interior solutions in Ladron-de-Guevara et al. (1999, p. 622-623). The proof is given here in appendix for completeness of exposition.

Equation (10) determines steady-state consumption as a share of household wealth. The interest rate, equation (11), is the sum of discount rate and GDP growth rate. Equations (12)-(13) determine the steady-state allocation of time. Combining (11) and (12)-(13) yields a steady-state interest rate equation:

$$r^2 - (\psi - \delta\theta)r + \delta^2\theta/\beta = 0, \tag{14}$$

A positive interdependence between the interest rate and the endogenous growth rate, equations (11) and (13), imply a multiplicity of steady state solutions. Let r_1 denote the lower root and r_2 the upper root of (14).[8] These roots are substantially different for empirically relevant parameter values. Ladron-de-Guevara et al (1999) emphasize qualitative differences between internal paths corresponding to these roots. However, the lower root of (14) is in many cases an implausible solution because the intensity of knowledge accumulation must be positive. For the steady state this implies that

$$\psi > \delta(1+\theta+\delta\theta/\beta r^*). \tag{15}$$

Here and henceforth asterisk relates to stationary growth. If, for instance, δ is sufficiently small, then (15) is fulfilled for r_2, but not for r_1. The case when the discount rate is low is important from theoretical and empirical points of view, and just in this case the lower

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[7] The steady state growth rate is obtained from (3) and (13): $g^* = \psi - (1+\theta)\delta - \theta\delta^2/\beta r^*$. The higher the discount rate or the elasticity of leisure, the lower the growth rate.

[8] Both roots are real and positive if $\psi - \delta\theta > 2\delta(\theta/\beta)^{1/2}$. Therefore, system (6)-(9) has two stationary states assuming either ψ is high or δ and θ are low. The steady state corresponding to r_2 is saddle-path stable if $r_2 > \delta(\theta/\beta)^{1/2}$ (Trofimov (2003), p. 18). This condition is fulfilled for $r^* = r_2$ because otherwise $\psi - \delta\theta < 2\delta(\theta/\beta)^{1/2}$ (because $r_2 > (\psi - \delta\theta)/2$) and (14) does not have real roots. Consequently a unique equilibrium path exists for $r^* = r_2$.

internal solution has to be ruled out.[9] In what follows we consider only an upper root of (14) and set $r^* = r_2$.

3. The global club economy

We extend the closed economy model to a world economy with N countries and free capital mobility. As in Lucas (1993, p. 254-255) and Barro & Sala-i-Martin (2001, p. 96-101), the global economy model contains only one final good, but international trade in goods can still take place. This mediates intertemporal exchange and allows for divergence of domestic investment and saving.

Households in country j are endowed initially with human capital h_{j0} and a stock of internationally tradable assets a_{j0}. Trade in the global capital market is opened at an initial time moment and capital is reallocated instantaneously across countries according to the marginal return on production investment. Initial production capital k_{j0} located into country j, therefore, differs from initial stock of assets a_{j0} in this same country. National capital markets become fully integrated in the sense that domestic households own a share of the global stock of capital and global investors own all domestic capital.

Countries have the same production technology, household preferences, population size (normalized to 1), but differ in initial stocks h_{j0}, a_{j0} and the productivity of growth-generating sector ψ_j which indicates the quality of institutions. Countries are ranked according to ψ_j, so that $\psi_1 \geq \psi_2 \geq \ldots \geq \psi_N$. The problem of country j household is:

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[9] We do not rule it out a priori but keep in mind that it is most likely an economically irrelevant solution for reasonable parameter values. As computations in Ladron-de-Guevara et al (1999, p. 619-620) demonstrate, (15) holds for r_1 for quite a narrow domain of the model parameters (for the log utility), and the steady state corresponding to r_1 is not optimal for the model with no adjustment costs. In many cases this state is unstable.

$$\max_{c_j, l_j, e_j, u_j, a_j, h_j} \int_0^{\infty} e^{-\delta t} [\ln c_j + \theta \ln l_j] dt, \tag{16}$$

$$\dot{a}_j = ra_j + wu_j h_j - c_j, \tag{17}$$

$$\dot{h}_j / h_j = \psi_j e_j, \tag{18}$$

$$u_j + e_j + l_j = 1, \tag{19}$$

$$e_j \geq 0, \tag{20}$$

The budget constraint (17) represents an accumulation of financial assets bringing return *r*. The term $ra_j + wu_jh_j$ in (17) is GNP per capita.

At any moment total assets and production capital are equal for countries – members of the capital market club:

$$\sum a_j = \sum k_j \tag{21}$$

where $j \in Club \subseteq \{N\}$.The marginal product of capital is equalized across countries, $r = \partial y_j/\partial k_j$ or

$$k_j = (r/\alpha)^{-1/(1-\alpha)} u_j h_j. \tag{22}$$

The wage rate is also equalized due to identity of technologies and linear homogeneity of production: $w = \partial y_j/\partial(u_jh_j) = (1-\alpha)(r/\alpha)^{-\alpha/(1-\alpha)}$.

Let $\varphi_j = y_j / \sum_{k \in Club} y_k$, $x_j = c_j/a_j$ and $z_j = a_j/k_j$ denote country *j* 's share in total output, consumption rate and capital position. We do not impose any constraint on the sign of net foreign assets held by households as this variable may take negative values along the transition path. In what follows, we impose constraints on the model parameters forbidding variables a_j, x_j, and z_j to be negative in the steady state.

Proposition 2[10]. *Equilibrium dynamics of the club economy are defined by the interest rate r and a set of country-specific variables* $\{k_j, z_j, \varphi_j, x_j, l_j, u_j, e_j\}$ *satisfying (18), (19), (22) and*

$$\dot{x}_j / x_j = x_j - \beta r / z_j - \delta \tag{23}$$

$$\dot{r} / r = \beta (\psi_j (1 - l_j) - r) \tag{24}$$

$$\dot{l}_j / l_j = \psi_j u_j - \delta \tag{25}$$

$$u_j = \beta r l_j / \theta x_j z_j \tag{26}$$

$$\sum z_k \varphi_k = 1 \tag{27}$$

Equilibrium dynamic equations (23)-(26) are essentially similar to those for the closed economy. Market-clearing condition (27) follows from (21), (22) and specifies interconnections between national economies. According to (27), the average capital position weighted by shares of countries in global output is unity. These shares are determined on the basis of the knowledge accumulation equations (18).

Suppose that $n \leq N$ countries are members of the global club economy. Then the dynamic system for the global economy consists of $7 \times n+1$ variables and $7 \times n+1$ equations. Interest rate equation (24) is compatible for all countries if the following n-1 conditions fulfill:

$$\psi_j(1- l_j) = \psi_k(1- l_k), \tag{28}$$

$j \neq k$. The number of equations is unchanged since n equations (24) transform to the unique interest rate equation. Constraints (28) imply that leisure is adjusted in each country in such a way that interest rate changes are identical for all countries at any time.

To clarify the mechanism of interest rate equalization, note that the corresponding equation (24) is obtained by taking log derivatives over the consumption-leisure equation:

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[10] The proof is straightforward. Dividing (17) by a_j and subtracting it from the Euler equation yields (23): $\dot{c}_j / c_j - \dot{a}_j / a_j = x_j - \beta r / z_j - \delta$. Equations (24)-(26) are derived similarly to (7)- (9).

$c_j/l_jh_j = w/\theta$ (this follows from (26) because $w = \beta rk_j/u_jh_j$.) According to this, marginal gain in household value from an increase of leisure is equal to marginal loss of this value from a decrease of labour input in production. As a result, the ratio of consumption to qualified leisure measured by the product of leisure and human capital is proportional to the wage rate. Since the wage rate and consumption grow at the same rate in all participating countries, qualified leisure must also grow at the same rate. From (18), (25), the growth rate of qualified leisure is $\dot{h}_j/h_j + \dot{l}_j/l_j = \psi_j u_j - \delta + \psi_j e_j = \psi_j(1-l_j) - \delta$. These growth rates are equal across countries due to the interest rate equalization implying, as seen from (28), that households in countries with superior institutions spend more time on leisure than households in other countries.

Leisure is a variable with law of motion (25) and is adjusted through variations in capital position z_j entering (26). Since changes of leisure in n-1 countries are predetermined by (28), capital position and consumption rate is found for these countries from equations (23) and (25)-(26). Given that the effect of each country on the interest rate determination is negligible, these equations imply that an increase of leisure reduces capital position and raises consumption rates in a country with superior institutions. Adjustments of variables z_j and x_j in n-1 countries therefore ensure interest rate equalization. The equilibrium interest rate and the capital position for a "residual" country is found from the interest rate equation (24) and market-clearing condition (27). Allocation of household time and consumption rates in n-1 countries is determined by the allocation of time and consumption rates in the residual country that is assumed to be the most advanced (country 1). The global capital market preconditions, thereby, close interdependence of household behavior over the global economy and its dependence on characteristics of the most advanced economy.

The causal relationship of this cross-country instantaneous adjustment is summarized in the following way. Interest rate equalization implies unification of labour price across countries. Due to immobility of labour force, households in each country have to adjust time structure to the wage rate through changes in capital positions and consumption rates. Countries with superior institutions are more attractive for capital, and, hence, households in these countries can increase consumption rates and leisure and reduce time for production and knowledge creation. On the contrary, households in economies with inferior institutions have to spend less time on leisure and more time on creative activities. Such a cross-country process of household time structure adjustment ensures equalizations of long-term growth rates across countries.

The cornerstone of this mechanism is leisure preference. The leisure-in-utility assumption proves to be important for a realistic description of the global economy dynamics. Suppose that $\theta = 0$, as is the case in the Lucas (1988) model. Then $l_j = 0$, and (24) becomes incompatible for countries with different ψ_j: $\dot{r}/r = \beta(\psi_j - r)$. A straightforward extension of the Lucas model to the heterogeneous global economy implies a counterfactual outcome when all production capital concentrates initially in the most advanced countries with maximal ψ_j, and interest rate is constant: $r \equiv \psi_1$. There is no mechanism in this model eliminating the gap between r and ψ_j for all other countries.

The balanced growth path of the club economy satisfies (19), (26), (27) and

$$x_j = \beta r/z_j + \delta, \tag{29}$$

$$r = \psi_j(1 - l_j), \tag{30}$$

$$u_j = \delta/\psi_j. \tag{31}$$

Equations (29)-(31) are similar to (10)-(12), determining the balanced growth path for the closed economy. From (31) and the capital allocation condition (22), the country share in total output is

$$\varphi_j = \frac{h_{j0}/\psi_j}{\sum h_{k0}/\psi_k}, \tag{32}$$

where h_{j0} relates to the balanced growth path.

The interest rate equation is drawn from (26)- (27), and (29)-(31) as

$$r^2 - (\overline{\psi} - \delta\theta)r + \delta^2\theta/\beta = 0, \tag{33}$$

where $\overline{\psi} = \sum \varphi_j \psi_j$ is an average productivity of knowledge creation. Equation (33) is analogous to (14) and has the similar properties. The steady state leisure is $l_j^* = (\theta\delta + z_j^*\omega)/\psi_j$ where $\omega = \theta\delta^2/\beta r^*$, therefore, $\psi_j(1 - l_j^*) = \psi_j - \theta\delta - z_j^*\omega$. The term $\psi_j(1-l_j^*)$ is identical across countries if the following equation fulfills for z_j^*: [11]

$$z_j^* = 1 - (\overline{\psi} - \psi_j)/\omega, \tag{34}$$

Hence, z_j^* is increasing in ψ_j, and $z_j^* <(>) 1$ for countries with $\psi_j <(>) \overline{\psi}$. This means that countries with lower than average quality of institutions are net lenders in the steady state, while other countries are net debtors.

From (34), the steady-state growth rate is the same across countries, $g^* = \psi_j e_j^* = \overline{\psi} - (1+\theta)\delta - \omega$ [12], and positive if

$$\overline{\psi} \geq (1+\theta)\delta + \omega. \tag{35}$$

This condition coincides with (15) obtained above for the closed economy. It relates to the global club economy and requires that the average productivity of knowledge creation is sufficiently high.

4. Membership in the club

It is reasonable to impose restrictions on the model parameters ensuring that x_j and z_j are non-negative in the steady state for all countries. These variables are positive in autarky

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[11] Indeed, $\psi_j(1-l_j^*) = \psi_{j-1}(1-l_{j-1}^*)$ implies that $z_j^* = z_{j-1}^* + (\psi_j - \psi_{j-1})/\omega$. Iterating terms yields $z_j^* = z_1^* + (\psi_j - \psi_1)/\omega$, $j = 2,\ldots,N$. Inserting z_j^* into (27) we have: $z_1^* = 1 - (\overline{\psi} - \psi_1)/\omega$, and this yields (34).

[12] $\psi_j e_j^* = \psi_j - (1+\theta)\delta - z_j^*\omega = \psi_j - (1+\theta)\delta - \omega + (\overline{\psi} - \psi_j) = \overline{\psi} - (1+\theta)\delta - \omega.$

while they may be negative for some countries in the open economy case. Such a long-run outcome means an exponential running of non-collateralized foreign debt, and should be ruled out as an implausible case. Restriction on the sign of household assets, $z_j^* \geq 0$, implies that

$$\psi_j \geq \overline{\psi} - \omega, \tag{36}$$

for $j \in Club$.[13] This inequality holds if either the productivity of knowledge creation in country j is not small compared to the world average or the elasticity of leisure and the discount rate are quite high.

Constraints (35) and (36) constitute conditions of economic integration via the global capital market. They constrain preference parameters δ and θ in the opposite way. Worldwide endogenous growth is positive if individuals are patient and inclined to creative activity. Contrarily, countries do not accumulate exponential debt if individuals are impatient and leisure loving. Unlike the closed economy case, consumer preferences ambiguously influence conditions of global growth.

Clearly, condition (36) holds for countries $j = 1, \ldots, n$, and does not hold for $j = n+1, \ldots, N$ thus dividing the world into two groups of countries. Less advanced economies $n+1, \ldots, N$ are unable to join the club without running exponential debt, and therefore (36) is fulfilled only for members of the club. To show how to calculate the threshold number n, note that the average productivity of knowledge creation for club-members $\{1, \ldots, n\}$ depends on this number: $\overline{\psi}_n = \sum_{j=1}^{n} \varphi_j \psi_j = \sum_{j=1}^{n} h_{j0} / (\sum_{j=1}^{n} h_{j0} / \psi_j)$. The long-run interest rate also depends on n because it is an increasing function of $\overline{\psi}_n$: $r^* = r_2 = r^*(\overline{\psi}_n)$. Constraint (36) is represented as an inequality on n:

[13] This requirement is stronger than the standard no-Ponzi-game condition related to variable a_j and fulfilled for the balanced growth path with negative z_j^*.

$$\psi_n \geq \overline{\psi}_n - \theta\delta^2 / \beta r^*(\overline{\psi}_n) \tag{37}$$

This constraint is not fulfilled for n+1 and it becomes very restrictive if the discount rate is low. In other words, only the most advanced economies can join the club if households are quite patient. Intuitively, if the discount rate is small, the long-term share of leisure is small either, and there is little room for the adjustment of household time structure without running exponential debt.

A numerical example illustrates this inference of the model. Suppose that the initial level of human capital is the same across all countries[14], $h_{10} = h_{20} = \ldots = h_{N0}$. Let $\delta = 0.025$, $\theta = 1$, $\beta = 2$, $N = 20$, and ψ_j be distributed uniformly according to the rule: $\psi_{j+1} = \psi_j - 0.005$, and $\psi_1 = 0.15$. Then, from (32), $\overline{\psi}_n = n / \sum_{j=1}^{n} 1/\psi_j = 0.1448$ for $n = 3$. This is a threshold number because $r^*(\overline{\psi}_3) = 0.0632$ and the right-hand side of (37) is equal to $0.1399 < 0.14$. The threshold level of productivity is $\psi_3 = 0.14$, and only three countries from twenty can become the club members satisfying (36). Notice that the threshold level of ψ_j sufficient for positive endogenous growth in autarky is $\psi_{20} = 0.055$, and that all 20 countries are able to meet condition of such growth (14).

Another numerical example demonstrates the effect of coordinated entry of less advanced economies into the global capital market. Parameters δ, θ, β are as above, $h_{10} = \ldots = h_{N0}$, and there are 11 countries such that country 1 is a leader, $\psi_1 = 0.15$, and all other countries are backward, $\psi_j = 0.1$. No one backward economy can join the club unilaterally thus establishing a two-country club with the leader. In such a case $n = 2$, $\overline{\psi}_2 = 0.12$, $r^*(\overline{\psi}_2) = 0.0494$, and the right-hand side of (37) is equal to $0.1137 > 0.1$. But if all

[14] This is consistent with the statement that h_{j0} relate to the balanced growth path because this path can be defined by a proper choice of initial allocation of assets across countries.

countries establish the global club simultaneously, then n = 11, $\overline{\psi}_{11}$ = 0.1031, $r^*(\overline{\psi}_{11})$ = 0.0403, and the right-hand side of (37) is equal to 0.0953 < 0.1. This example demonstrates that multilateral decisions can facilitate integration within the capital market club.

Capital market imperfections are often viewed as the main obstacle to economic integration. Our model's barriers to integration stem from a structural heterogeneity of countries rather than exogenous borrowing constraints. If we imposed such a constraint, for example $z_j^* \geq z_{min}$, condition (36) would imply a more stringent restriction on the variation of ψ_j across countries. If, on the other hand, we did not restrict the sign of z_j^* we would have to deal with the weaker condition that leisure is positive, thus, implying that $\psi_j > \overline{\psi} - \omega - \delta\theta$. This restriction on the model parameters is qualitatively similar to (36).

Alternatively, one could introduce exogenous minimal level of leisure l_{min} into the household utility function thereby imposing a stronger lower constraint on leisure: $l_j \geq l_{min}$. In fact, (36) is equivalent to $x_j^* \geq 0$ or $l_j^* \geq \theta u_j^*$ since $x_j^* = \delta l_j^*/(l_j^* - \theta u_j^*)$. Forbidding exponential debt expansion implies constraining the steady state leisure by a minimal value proportional to the intensity of production. This condition holds automatically for the closed economy, as seen from (12), (13).

Essentially, the diversity of countries in ψ_j is necessary for violation of (36). This condition is fulfilled if countries have the same $\psi_j = \psi$ but differ in preference and technology parameters δ, θ or β. In this case, countries have different ω_j, and the steady-state capital positions are $z_j^* = \omega_j/\overline{\omega} > 0$ where $\overline{\omega}$ is the average of ω_j.[15] Consequently, the diversity of countries in growth-supporting institutions is a crucial barrier to membership in the club rather than diversity in preference and technology.

[15] Condition $\psi(1-l_j^*) = \psi(1-l_1^*)$ implies $z_j^* = z_1^* \omega_j / \omega_1$. Inserting it into (27) yields: $z_1^* = \omega_1 / \overline{\omega}$ and, hence, $z_j^* = \omega_j/\overline{\omega}$.

Delaying trade liberalization reduces the chance for a less advanced country to join the capital market club unilaterally. Economies grow at different rates before opening its capital market at time $t = 0$. If this is delayed indefinitely, the gap between initial stocks of human capital widens and the limit $\overline{\psi}$ approaches ψ_1. Thus, the longer the pre-liberalization period, the higher the weight of advanced countries in $\overline{\psi}$, and the larger the gap $\overline{\psi} - \psi_j$ becomes for backward countries. As a result, the number of autarkic economies able to meet (36) tends to reduce in time.

5.Transition within the club

The issue of transition is relevant since the radical liberalization of the global capital market has occurred not long ago. If, presumably, participating countries had been growing along the autarky balanced growth paths before capital market opening, then, after this happened, they switched to a trading transition path. This section focuses on the transitory effects of instantaneous cross-country capital reallocation.

We have shown above that if the discount rate is quite low, only countries with nearly the same quality of institutions can unilaterally join the club. From this we may analyze the transition dynamics of the global club economy for the simplest case when ψ_j is the same for all countries, $\psi_j = \psi$, and the national economies differ in their initial factor endowments a_{j0} and h_{j0}.[16] The difference in initial conditions is irrelevant for the balanced growth path. Also, the steady state is the same for all countries as seen from (34) then implying that $z_j = 1$. Nevertheless, their transition dynamics may differ due to the diversity of initial endowments.

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[16] Another reason is that the dynamic system for heterogeneous economies is hardly tractable analytically. In the case of 2 countries with different ψ_j we obtain a highly non-linear system of dimension 5.

Interest rate equalization implies, in this case, identity of household time structure for all countries at any time. This is seen from conditions (28) and (25) implying that $l_j = l_k \equiv l$ and $u_j = u_k \equiv u$ for all j and k, as well as ensuring the identity of dynamic equations for the interest rate and leisure. From (19) $e_j = e_k \equiv e$ for all j and k, and at any time human capital grows at the same rate in all countries. Production capital and output also grow at the same rate since k_j is proportional to uh_j, and GDP growth rates converge instantaneously at the initial time. Consumption growth rates are also identical along the transition path. Equilibrium paths of countries differ only in consumption rates and capital positions.

Denote the aggregate global consumption rate as $\bar{x} = \sum c_j / \sum a_j$, $j \in$ *Club*. The global economy trajectory is described by aggregate variables $\bar{x}$, r, l, and u.

Proposition 3. An aggregate equilibrium trajectory of the global club economy satisfies the autarky dynamical system (6)-(9) with x substituted for $\bar{x}$.

The aggregate trajectory is saddle-path stable (see footnote 7) and is determined by initial aggregate consumption rate $\bar{x}_0$, leisure l_0 and interest rate r_0. This choice is predetermined by initial aggregate knowledge-asset ratio $\eta_0 = \sum h_{j0} / \sum a_{j0}$, $j \in$ *Club*. The equilibrium interest rate and production intensity solve (22) and (26), implying an equation for the surface of initial values in ($\bar{x}$, r, l) space:

$$\bar{x} = (\eta_0 \beta / \theta) r \sigma(r) l, \tag{38}$$

where $\sigma(r) \equiv (r/\alpha)^{-1/(1-\alpha)} = k_j / uh_j$ is the input structure of production identical for all countries.

An equilibrium trajectory for country j differs from the aggregate trajectory in consumption rate and capital position, because the allocation of household time is determined by the global economy dynamics. The relationship for the surface of initial values in (x_j, r, l) space is similar to (38):

$$x_j = (\eta_{j0}\beta/\theta) r\sigma(r) l. \tag{39}$$

where $\eta_{j0} = h_{j0}/a_{j0}$ is the knowledge-asset ratio for country j. The national economy path is defined by variables r, l, u, x_j, z_j which satisfy (22)-(26).

The phase diagram for the global and national economies is depicted in figure 1 portraying the phase space (x, r, l) where $x = \bar{x}$ or x_j. All trajectories converge to the same stationary point G, but begin from different initial points.[17] The aggregate equilibrium trajectory begins from an initial point O, an intersection of the saddle path with the surface of initial values S as defined by (38). This point determines the initial values r_0 and l_0. The national economy trajectory begins from an initial point $O_j = (x_{j0}, r_0, l_0)$ belonging to surface S_j as defined by (39) and initial ratio h_{j0}/a_{j0}. Initial point O_j determines x_{j0} and z_{j0} for each national economy. Projections of country-specific trajectories on plane (l, r) coincide with the projection of the aggregate trajectory on this plane.

As seen from the figure, initial reallocation of capital within the club divides its members into two groups. Leading countries with higher than average initial ratio of knowledge to assets, $\eta_{j0} > \eta_0$, have higher than average consumption rates along the transition path, whereas these rates in other countries are lower than average. The former countries are net debtors, while the latter ones are net creditors. Such a pattern of cross-country interdependence forms a type of global economy with trade in capital, where the growth of consumer demand and production investment in leading national economies temporarily serves a driving force for the rest of the world.

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[17] The disaggregated global economy is represented by a 3+n-dimensional dynamic system including 3 equations for the aggregate variables $\bar{x}$, r, l and n consumption rate equations $\dot{x}_j / x_j = (1 - \beta r/\bar{x})x_j - \delta$ obtained from the consumption rate equation for $\bar{x}$ similar to (6), and equations (23), (26). Linearized around the steady state, this system has a block-diagonal structure if the countries are small, and one can ignore the link between each x_j and $\bar{x}$. Under the condition of proposition 3, the disaggregate global path is a saddle.

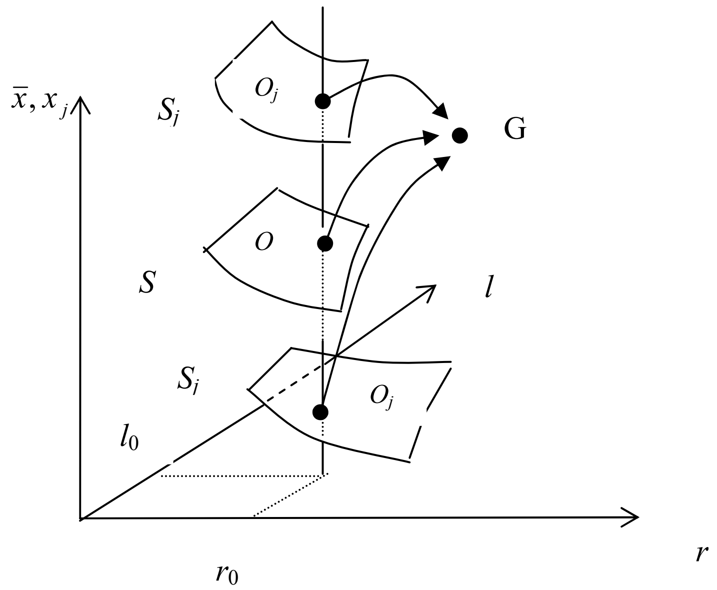

Figure 1. Global equilibrium path

6. Supplement: a small open economy

The case of a small open economy allows envisaging some aspects of integration of a single less advanced economy with the club of advanced economies. This supplement, first, examines a pattern of growth of a backward country having relatively poor institutions not satisfying the participation constraint (36). Second, it analyses the effect of knowledge transfers induced by foreign production investment on opportunities to join the club.

a. Outsider in the club

Less advanced economies that do not meet condition of membership in the club (36) can remain in autarky or join it in a very specific pattern when nothing is produced domestically, no production capital is allocated at home, human capital is not accumulated and households devote all time to leisure. Formally, this pattern of growth corresponds to a corner solution of the household problem (16)-(20) arising if constraint (20) forbidding negative efforts in knowledge creation is binding along the whole equilibrium path.[18] As shown in Appendix, an economy growing exogenously and trading in the world capital market transforms to the economy with zero physical capital and production. The steady state ratio of assets to capital is infinity, and the steady state consumption rate equals δ. Per capita assets and consumption grow in the steady state at the worldwide rate $g^* = r^* - \delta$.

Suppose that growth of the global economy is stationary and consider the transition of a small open economy to this steady state. The growth rate differential is $\Delta g = g^*$ because $e = 0$. The country subscript is omitted here and in what follows. As shown in Appendix, the consumption-capital ratio $\xi = c/k$ satisfy

$$\dot{\xi} / \xi = \Delta g(1 + (\theta / \beta r^*)\xi) . \tag{S1}$$

This equation is solved explicitly as $\xi = \xi_0 e^{\Delta gt} / [1 + (\theta / \beta r^*)\xi_0 (1 - e^{\Delta gt})]$, where ξ_0 is the initial consumption rate. This trajectory is explosive as ξ approaches infinity in finite time $T = ln(1+\beta r^*/\theta\xi_0)/\Delta g = - ln l_0 / \Delta g$. The transition period is inversely related to initial leisure and the growth differential.

Intuitively, such a regime of long-run growth is associated with an economy of a *rentier* type in the sense that households become pure financial investors holding foreign assets. Such a regime of growth is impossible in the closed economy. But it does exist, and may be welfare preferable to productive growth in the economy trading in the world capital market.[19] On the one hand, there are a lot of examples of very poor countries where

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[18] Such growth paths in the Lucas-Uzawa framework has been considered earlier by Rebelo (1991), Caballe & Santos (1993), Goodfriend & McDermott (1995), Ladron-de-Guevara et al. (1999), Trofimov (2003).

[19] Trofimov (2003) has shown that if households are quite patient, the rentier regime is selected by a country able to integrate and grow on endogenous basis. There is no positive connection between "thriftiness" and

national elite has been accumulating large volumes of international assets. On the other hand, large stocks of internationally tradable natural resources in resource-rich economies are equivalent to stocks of internationally tradable assets. Patterns of growth and specialization based on raw material trade correspond to the process of transformation of these economies to the rentier type.

b. Knowledge transfers

We have ignored knowledge transfers that constitute a factor of growth on the national level and facilitate economic integration. Lucas (1993) introduced positive spillover effects into the global economy model by assuming that the world stock of knowledge is a factor of knowledge production at home. In this case an economy with a stock of knowledge lower (higher) than the world average grows faster (slower) than the world economy. This assumption implies convergence of growth rates across countries, but this prediction does not fit fairly well the empirical regularities (see Durlauf & Quah (1999, p. 265-268)).

One can focus on another channel of knowledge spillovers induced by foreign capital inflows, or, more precisely, by foreign direct investment (FDI). FDI-induced accumulation of knowledge is normally interpreted in the sense of technology and know-how transfers, labor force training within subsidiaries or parent companies of multinationals, copying of advanced technologies by domestic firms in a country receiving foreign investment. These effects increase productivity in a host country and are especially pronounced if investments

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growth for an open economy: excessively thrifty households will prefer financial investment to productive activity and knowledge creation.

are made by advanced economies into less advanced ones.[20] We introduce investment-induced transfers simply as a positive feedback between total production investment and an increase of per capita stock of knowledge:

$$\dot{h} = \psi eh + \phi\dot{k}.$$

The last term in this equation refers to the link between production investment and an increase of knowledge. The model does not permit one to distinguish domestic and foreign production investment in the host economy and, therefore, FDI is treated as a part of total production investment. Coefficient $\phi \geq 0$ reflects the share of FDI in total investment and the effects of knowledge transfer. In percentage terms we have:

$$\dot{h}/h = \psi e + \varepsilon(u)\dot{k}/k \qquad \text{(S2)}$$

where $\varepsilon(u) = \phi u\sigma(r)$ is the elasticity of knowledge to production capital. It is increasing in production intensity in the host country and decreasing in the world interest rate.

At time $t = 0$ the economy opens to the world capital market, and the global economy is in the steady state with a constant growth rate g^* and interest rate $r^* = g^* + \delta$. The country's consumption starts to grow at t = 0 at constant rate $g^* = \overline{\psi} - (1+\theta)\delta - \omega$. The steady-state interest rate equation (30) for this country implies: $r^* = \psi(1-l) + \varepsilon(u)\dot{k}/k$. Unlike the case with no external effects of investment, the economy can adjust to the world interest rate through an inflow of production capital. Household time allocation is no longer predetermined at each instant by interest rate equalization.

As shown in the Appendix, the steady state allocation of household time is

[20] As R. Findlay (1978, p.1) points out, "while the "book of blueprints" in some abstract sense may be open to the world as a whole, even if one may have to pay a stiff price to look at some of the pages, new technology generally requires demonstration in the context of the local environment before it can be transferred effectively, and it is in this connection that the overseas production of major world corporations with their headquarters in the advanced countries has such a vital part to play."

$$u^* = \delta/\psi,\ e^* = (1- \varepsilon(u^*))g^*/\psi \qquad \text{(S.3)}$$

Consumption rate and the capital position are non-negative if $l^* \geq \theta u^*$ or

$$\psi \geq (1 - \varepsilon(u^*))(\overline{\psi} - \omega) + \varepsilon(u^*)(1+\theta)\delta. \qquad \text{(S.4)}$$

The elasticity $\varepsilon(u^*)$ is small if δ is small. In such a case (S.4) is close to (36) and the effect of knowledge transfers on this restriction is weak.

Consequently, if households are patient, investment-induced knowledge transfers only slightly widen the restriction forbidding participation in the world capital market. The low discount rate implies low intensity of production in the steady state and an insignificant effect of capital inflows on the accumulation of knowledge. This is consistent with the above conclusion that in itself patience is a virtue favoring economic growth. But being favorable for growth in autarky or in the club of economies, it narrows opportunities for integration of a less advanced country with this club.

7. Concluding remarks

We established that even if growth rates diverge for autarkic economies, they converge for countries trading in the world capital market. The property of club convergence in growth rates is closely related to the restrictions stipulating participation of economies in the capital market club and requiring their close similarity in terms of the quality of institutions. The human capital model of growth utilized in this paper ignores technological gaps that are essential for divergence of incomes and growth rates and are also relevant to the property of club convergence. Nevertheless, this framework allows to model in a simple way international linkages implied by cross-country capital flows and the adjustment of household behavior. The model demonstrates that this mechanism plays an important role in the emergence of the capital market club given that labor is an immobile factor of production.

In our view, the condition on the model parameters forbidding exponential accumulation of net external debt is relevant to the pitfalls of globalization. If some countries in the real world follow this pattern of growth, they sooner or later default on debt. One can interpret the condition of no-exponential net debt as prohibiting trade in the global capital market for countries strongly exposed to the risk of default. In line with this interpretation, financial crises can be viewed as unsuccessful attempts of integration with the developed world by countries with relatively poor growth-supporting institutions.

Characteristics of household behavior ambiguously affect growth of a closed and open economy. The autarkic long-run growth rate is high if households are quite patient, but there is no positive connection between thriftiness and growth for the open economy trying to enter the global capital market. This model prediction is in contrast with the conventional wisdom that openness and patience are the key conditions for economic success of a nation. Whatever the time preferences of households are, opening of the economy should occur in accord with institutional reforms stipulating its membership in the club of advanced nations.

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Appendix 1. Proof of propositions

Proposition 1. The Lagrangian for (1)-(5) is $L = lnc + \theta\, ln(1\text{-}u\text{-}e) + \lambda_1(y - c) + \lambda_2 \psi eh + \chi e$, where λ_1 and λ_2 are co-state variables related to (2) and (3), respectively and χ is a dual variable related to (5). For an interior solution $\chi = 0$, and the first order conditions are

$$1/c = \lambda_1 \tag{A1}$$

$$\theta/l = \lambda_2 \psi h \tag{A2}$$

$$\theta/l = \lambda_1(1\text{-}\alpha)(k/uh)^{\alpha}h. \tag{A3}$$

The co-state equations are

$$\dot{c}/c = r - \delta. \tag{A4}$$

$$\dot{\lambda}_2 = \delta\lambda_2 - (1-\alpha)(k/uh)^{\alpha} u\lambda_1 - \psi e\lambda_2. \tag{A5}$$

Combining (A2) and (A3) implies $(1\text{-}\alpha)(k/uh)^{\alpha} = \psi\lambda_2/\lambda_1$. Substituting this for (A5) yields

$$\dot{\lambda}_2/\lambda_2 = \delta - \psi u - \dot{h}/h. \tag{A6}$$

Taking log derivatives of (A2) yields $\dot{\lambda}_2/\lambda_2 = -\dot{l}/l - \dot{h}/h.$ Substituting this for (A6) implies (8). Dividing (2) by k and subtracting (A4) from (2) yields (6):

$\dot{c}/c - \dot{k}/k = r - \delta - r/\alpha + x = x - \beta r - \delta.$

To derive (7) utilize (A4), (3), (8), equation $r = \partial y/\partial k$ and the log derivatives of (A3) $\dot{r}/r = \beta(\dot{l}/l + \dot{\lambda}_1/\lambda_1 + \dot{h}/h) = \beta(\dot{l}/l - \dot{c}/c + \dot{h}/h) = \beta(\psi(1-l) - r)$. To derive (9) utilize (A1) and rearrange (A3).

Proposition 3. Equations similar to (7) and (8) follow from the identity of time allocation between countries. To derive equation similar to (6), sum up (A4) and (17) across countries yielding $\dot{C}/C - \dot{A}/A = r - \delta - (r + w\sum uh_j / A - C/A)$, where $C = \sum c_j$, $A = \sum a_j$. Utilizing (22) we yield $w\sum uh_j / A = (1-\alpha)(r/\alpha)^{-\alpha/(1-\alpha)}(r/\alpha)^{1/(1-\alpha)}\sum k_j / A = \beta r$ and obtain equation $\dot{\bar{x}}/\bar{x} = \bar{x} - \beta r - \delta$. Summing (26) across countries yields equation $u = \beta rl/\theta\bar{x}$ since $x_j z_j$ is the same across countries.

Appendix 2. Proof of statements in Supplement

Equation (S.1). Constraint (20) is binding for the corner solution, and the first order conditions are (A1), (A3) and $\theta/l = \lambda_2\psi h + \chi$. The co-state equations are (A4), (A5). Equilibrium dynamics of the open economy satisfy (23) and

$$\dot{r}/r = \beta\frac{\delta + u\dot{\xi}/\xi - r}{1+\beta u}, \tag{A7}$$

$$u_j = \frac{\beta r}{\beta r + \theta\xi}. \tag{A8}$$

Equation (A8) is derived as (9) was. Differentiating (A8) and rearranging terms yields $\dot{u}/u = l(\dot{r}/r - \dot{x}/x - \dot{z}/z)$. Therefore differentiating (22) and utilizing (17), (23) yields

$$\dot{r}/r = (1-\alpha)(\dot{u}/u + \dot{h}/h - \dot{k}/k) = (1-\alpha)[l(\dot{r}/r - \dot{x}/x - \dot{z}/z) + \dot{z}/z - \dot{a}/a] =$$
$$(1-\alpha)[l(\dot{r}/r - \dot{x}/x - \dot{z}/z) + \dot{z}/z - r - \beta r/z + x] = (1-\alpha)[l\dot{r}/r + u(\dot{x}/x + \dot{z}/z) + \delta - r].$$

This implies (A7). Combining (A7) and (A8) for $r = r^*$ yields (S.1).

Equations (S.3) The growth of the national economy is expressed by equations

$$\dot{z} / z = \dot{\xi} / \xi - \xi / z + \beta r^* / z + \delta \qquad (A9)$$

$$\dot{\xi} / \xi = g^* + (\psi(1-l) - r^*) / \varepsilon(u) \qquad (A10)$$

and (25)-(26). Here (*A9*) is consumption rate equation (23) expressed in variables ξ and z, (*A10*) is the interest rate equation (24) for $r \equiv r^*$. The steady state allocation of household time is $u^* = \delta/\psi$, $e^* = (1- \varepsilon(u^*))g^*/\psi$ since $r^* = g^* + \delta$. The steady-state capital position and consumption rate are $z^* = \dfrac{\beta r(l^* - \theta u^*)}{\theta \delta u^*}$, $x^* = \dfrac{\delta l^*}{l^* - \theta u^*}$ because $\xi^* = \beta r^* + \delta z^*$ and $\xi^* = \beta r^* l^* / \theta u^*$.