THE TERMS OF TRADE AND THE RELATIVE PRODUCTIVITY OF TRADABLES

by Georgiy Trofimov
The Institute for Financial Studies

Mail address: 143082, Zhukovka, Odintsovsky district, Moscow Region, Russia
E-mail: gerat@ifs.ru
Telephone: 7-495-710 78 10,
Fax: 7-495-710 78 11

October 2006

Abstract. The paper examines positive interconnections between terms of trade and cross-country relative productivities. It presents a model of bilateral intraindustry trade and productivity growth featuring direct and indirect terms-of-trade effects on revenues of firms. The former is negative and enforces domestic firms to invest in new technology, while the latter is positive and encourages entry of domestic firms. Interaction of these effects stipulates a positive dynamic link between the relative productivity and the terms of trade generating empirically relevant patterns of long-run productivity dynamics across countries: mean-reversion of relative productivity and conditional convergence of outputs per capita. A leading economy is shown to gain in terms of enhanced market share after opening trade, but may stagnate because of the terms of trade improvement. A backward economy gains from higher initial productivity gap, while the leading economy loses.

1. Introduction

The interconnection of cross-country relative prices and productivities is among the core issues of trade and growth theory. Traditionally, the causal effect of productivity on the national price level has been in the main focus of research. The widely addressed phenomenon of post-war international dynamics is the Balassa-Samuelson effect (Balassa 1964, Samuelson 1964). It consists of the fact that the relative price level is normally higher in the technologically advanced countries and tends to increase for the countries exhibiting rapid productivity growth in tradables. But there is also substantial evidence suggesting that, besides raising the general price level, productivity gains of a country tend to improve its terms of trade (Alquist, Chinn 2002, Debaere, Lee, 2004, Dedola et al. 2004, Corsetti 2004). This effect is less clear theoretically, as compared to the Balassa-Samuelson effect which is explained relying on the premise of persistent productivity gap between tradables and non-tradables. A puzzling question is why do productivity gains obtained by a country tend to improve its terms of trade in spite of decreasing production costs and increasing supply of tradables?

A converse causality manifests in the influence of the terms of trade on relative productivity dynamics. Cross-country data demonstrate a positive effect of the terms of trade on the long-run growth, implying a reduction of output gap due to an improvement of the terms of trade for the lagged economy. For instance, Barro (1999) uses the terms-of-trade improvement as a control variable in the cross-country growth regression built for 98 countries and embracing the period 1960-90. The change of this variable, measured as the ratio of export to import prices, has a positive and strongly significant impact on the long-run growth of GDP per capita. Barro concludes from this finding that “an improvement in the terms of trade apparently does stimulate an expansion of domestic output” (Barro 1999, p. 30). But the long-term improvement of the terms of trade, once it accompanies productivity growth, tends to worsen competitiveness, thereby slowing down long-term growth. The question is in what way do the terms of trade may influence productivity growth in the long term, and how does this contribute to the tendency of productivity gaps reduction across trading countries?

This paper is addressing the issues related to the causal links between the terms of trade and the relative productivity of tradables. The basic intuition is that the set of traded goods need not be fixed at any time period. According to the conventional view, the relative productivity growth should, ceteris paribus, deteriorate the terms of trade. But the number of domestic goods and production firms may increase in response to the obtained productivity gain, raising home demand for factor services and hence factor prices. Under provisions of imperfect competition, the price of exportables will receive impetus to increase, despite productivity growth at home, thus tending to improve the terms of trade. The terms-of-trade improvement, in turn, affects the costs and benefits of investment in productivity growth decided by firms.

To examine the outlined causalities and the ensuing long term patterns of trade and growth, we suggest a simple two-country, one-factor model of productivity growth induced by bilateral trade. This is a dynamic model of monopolistic competition incorporating trade in differentiated goods and investment in technology improvement. For each firm the motive to invest is to make production cheaper than overseas, bringing about a reduction of the relative price for this firm and expansion of its revenue. The numbers of firms in both economies are determined endogenously at each period of time, and the terms-of-trade effects on these numbers are neglected by firms. A key property of the model is that a terms-of-trade improvement encourages entry of domestic firms and discourages entry of foreign firms. This effect causes an expansion of the domestic market share since it outweighs the negative direct effect of the terms-of-trade improvement accounted for by firms.

In trading equilibrium of the model there is a static and dynamic link between the terms of trade and the concurrent relative productivity. The static link is defined through the relative wage determination and may be negative or positive, depending on the value of intratemporal elasticity of substitution between goods. The relative wage is increasing in the domestic market share and hence in the terms of trade. This determines a static dependence between concurrent terms of trade and relative productivity, which is positive if the elasticity of substitution is low (below 2) and negative if it is high (above 2). A current productivity gain of the country affects ambiguously the current period terms of trade, because it can be translated into a larger or smaller rise of the relative wage. But as it turns out, the dynamic link between these variables is unambiguously positive, implying an improvement of the terms of trade in response to the lagged productivity gain. The reason is that this gain induces growth in the relative number of domestic firms in the current period causing increase of relative wage that outstrips current productivity growth.

The positive dynamic link is essential for the long-term pattern of productivity growth generated by the model. The initial relative productivity is low for the less advanced economy implying that the terms of trade in the first period are also low, and the relative productivity of this economy grows up. The obtained productivity gain, in turn, improves its terms of trade in the second period, implying growth of relative productivity at slower pace, and so on. The model, thus, exhibits mean-reverting dynamics of relative productivity and terms of trade, and global convergence to the steady state for a permissible domain of the model parameters. The pattern and speed of convergence depend on the intratemporal elasticity of substitution between goods. Convergence is monotonous for the elasticity below 2 and oscillating for this above 2. The terms-of-trade effect on the relative productivity is strong enough in the latter case to induce overtaking of the leading economy in one period of time. Although being intuitively clear, the case of alternating leadership among the national economies is likely to be empirically relevant only for very long historical periods.

On the contrary, the monotonous convergence property of the model is strongly supported by the abovementioned cross-country evidence of positive long-term dependence between GDP growth and terms-of-trade improvement. Accounting for the close link between total factor productivity and output per worker (Hall, Jones 1999), one may interpret this evidence as follows. Starting with low initial relative productivity, a less advanced economy faces low terms of trade at the beginning of transition path and obtains an advantage in growth of its tradable sectors. This economy, therefore, exhibits growth of both relative productivity and terms of trade along the development path, as suggested by the evidence and implied by the positive mutual interdependence of these variables in our model.

This feature of the model is relevant to the conditional convergence property of global growth (Barro 1991). According to it, countries with higher income per capita tend to grow slower than those with lower income, provided that measures of education and government policies are controlled for. The underlying theory for the closed economy is the neoclassical growth model with diminishing returns to production capital. For the global economy, the pattern of cross-border price movements, rather than diminishing returns to capital, may be crucial for global convergence of incomes, as has been shown by Ventura (1997) for a model of global capital accumulation. In line with this reasoning, the terms of trade are viewed in our paper as an important factor of cross-country productivity dynamics stipulating mean-reverting effects and conditional convergence over long horizon.

Our model in particular suggests that the processes of economic integration and convergence are two-sided in the sense that growth of the backward economy accelerates at the expense of slowing productivity growth of the leading economy. This view is in marked contrast to the tenet of diminishing returns to capital underlying the conditional convergence property for the world of autarkic countries. The emphasis is usually placed on the process of catching up by the less advanced economies. Less attention is given to the other side of convergence – the process of slowing down by the leading economies. The interior trading equilibrium, in which both countries exhibit positive growth, exists for our model only if the initial productivity gap is not too large. Otherwise trading equilibrium may only be asymmetric, that is based on the corner solution in which firms in the advanced economy are forced to stop investment because of too high terms of trade. The leading economy is stagnating for a while, until the time when the productivity gap is sufficiently narrowed for the conditions of positive growth to be fulfilled for both economies. This result suggests that slowdowns in growth, being suffered by some advanced economies during last decades, may be rendered, besides other factors, by trade liberalization and technological convergence with the less advanced economies.

The closely related subject concerns utility gains and losses from trade between leading and backward economies. This has been examined in the literature on trade and growth suggesting ambiguous inferences (surveyed, for instance, by Aghion and Howitt 1998, pp. 366-401). The welfare analysis has been focused merely on the issues of a trade-off between static and dynamic gains from trade and unfavorable specialization of the less developed country. In our model trade benefits both economies because it induces global growth and extends the number of varieties available to households. We, therefore, address a different issue, namely, of cross-country division of utility gains from trade and growth and its dependence on the size of initial productivity gap. The terms of trade of the lagged economy in our model deteriorate after opening trade, implying lower market share and national income, but higher initial growth rate. Conversely, the advanced economy gains from increased market share, but suffers reduced growth rate. Household utility is computed for the benchmark case of intratemporal elasticity of substitution equal to 2 and shown to be decreasing in the initial relative productivity. It means that the higher the initial productivity gap is, the more pronounced is the growth effect benefiting the lagged economy to a larger extent than the advanced economy. As a result, the backward economy gains relatively more from its backwardness than the leading economy from its leadership, if they are engaged in intraindustry trade.

Our model resembles the framework of trade and endogenous growth based on R&D and increasing specialization suggested by Grossman and Helpman (1990, 1991, 1995). In their model of bilateral trade with two final good sectors, the R&D sector supplies new varieties to the differentiated goods sector. Our model is of purely intraindustry trade with firms investing in productivity growth and the number of firms (varieties) determined at each period by free entry. Grossman and Helpman emphasize the effects of trade on labor allocation between research and production, and on the resulting intensity of technical change. Depending on factor endowments and dynamics of comparative advantages in these activities, trade has been shown to lead to patterns of specialization and growth reinforcing the position of the leading nation in the long run. Here we examine the effects of trade on growth under a different angle, placing emphasis on the dynamic link between terms of trade and relative productivity instead of dynamic comparative advantages.

Our paper is more closely related in its goals to the recent papers by Corsetti et al. (2005) and Acemoglu and Ventura (2002). The former explains why a productivity gain obtained by a country may result in the terms-of-trade improvement. This paper suggests a static model of bilateral trade with endogenous number of differentiated goods and monopolistic competition. It makes a distinction between productivity gains in manufacturing and creation of new firms affecting the terms of trade in the opposite way. Lower manufacturing costs lead to supply of domestic goods at a reduced international price, but lower entry costs imply lower operating profits resulting in a larger array of goods in the market and a terms-of-trade improvement. Unlike Corsetti et al. (2005), we assume endogenous movement of productivity gains in manufacturing and obtain positive causal effect of these gains on the terms of trade.

The paper by Acemoglu and Ventura (2002) shows that, in the absence of diminishing returns to production and technological spillovers, international trade leads to a stable world income distribution. Convergence of incomes is shown for a global economy model of capital accumulation with continuum of countries and perfect competition among exporting firms (in the basic version of their model). Countries accumulating capital faster than average expand their exports faster and experience declining export prices. This depresses the rate of return to capital and discourages further growth. In our model, as in their, international spillovers of technology are absent and returns to investment are non-decreasing. Our model departs from the Acemoglu and Ventura's in several respects: the world consists of two countries, there is no capital accumulation, and firms are monopolistic competitors investing in productivity improvement. Our model predicts, similarly to their, that trade ensures convergence of productivities, but this inference rests upon a different mechanism to be discussed in what follows.

The paper is organized as follows: Section 2 presents the model; properties of trading equilibrium are analyzed in Sections 3 and 4; the model dynamics are examined in Section 5 also containing an informal discussion of the results. Section 6 considers asymmetric trading equilibrium with stagnation, and Section 7 is devoted to computation of utility gains and losses. Proofs of propositions and figures are collected in Appendix.

2. The model

There are two economies trading in the common market for goods and upgrading productivities through investment in technology by firms. Each firm is producing a single differentiated good and engaged in monopolistic competition with other firms. Production sector in each economy supplies a spectrum of differentiated goods. The number of goods is endogenous and determined at each time period, which is sufficiently long for excess monopoly profits to be eliminated by entry of new firms.

Households in both economies have the same preferences. They exchange assets and liabilities to provide consumption smoothing in time. Bilateral trade is balanced on the indefinite time horizon. There is no production capital in the model, and all assets bring a constant real return equal to the household discount rate. Labor is internationally immobile, and a household supply a unity of labor to the local production sector. Countries are of equal size, and in each the amount of labor is equal to the number of households and normalized to unity.

2.1 Households

The integral household utility is logarithmic:

$$U = \sum_{t=1}^{\infty} \beta^t \ln C_t \qquad U^* = \sum_{t=1}^{\infty} \beta^t \ln C_t^* \tag{1}$$

where C_t and C_t* is an aggregate consumption index at time period t for domestic and foreign households (throughout, the asterisk relates to the foreign country), β is the discount factor. Aggregate consumption indices indicate CES preferences over domestic and foreign goods:

$$C_t = \left(\int_0^{n_t} c_{dt}(j)^{1/\theta} dj + \int_0^{n_t^*} c_{it}(j)^{1/\theta} dj \right)^{\theta}, \; C_t = \left(\int_0^{n_t^*} c_{dt}^*(j)^{1/\theta} dj + \int_0^{n_t} c_{it}^*(j)^{1/\theta} dj \right)^{\theta}, \tag{2}$$

where $c_{dt}(j)$ and $c_{it}(j)$ ($c_{dt}*(j)$ and $c_{it}*(j)$) is consumption of domestic and imported goods at home (abroad), j is the index of goods, $\theta=\sigma/(\sigma-1)$, and $\sigma>1$ is the intratemporal elasticity of substitution across goods. The number of goods produced at home and abroad is n_t and n_t*, respectively. Each good is supplied by a single firm to the common market without trading costs.

Domestic and foreign households face intertemporal budget constraints:

$$\sum_{t=1}^{\infty}\beta^t P_t C_t=\sum_{t=1}^{\infty}\beta^t Y_t\,,\quad \sum_{t=1}^{\infty}\beta^t P_t C_t*=\sum_{t=1}^{\infty}\beta^t Y_t* \tag{3}$$

where Y_t (Y_t*) is domestic (foreign) household current income, P_t is a common aggregate price index:

$$P_t=\left(\int_0^{n_t}p_t(j)^{-1/\mu}dj+\int_0^{n_t*}p_t*(j)^{-1/\mu}dj\right)^{-\mu}, \tag{4}$$

where $\mu=1/(\sigma-1)$, $p_t(j)$ and $p_t*(j)$ are price indices of domestic and foreign goods, respectively.

Goods and firms are symmetric in each country, and their indices j are omitted in what follows. At any period of time households choose the bundle of domestic and imported goods subject to a one-period budget constraint:

$$n_t p_t c_{dt}+n_t*p_t*c_{it}=E_t\,,\qquad n_t*p_t*c_{dt}*+n_t p_t c_{it}*=E_t* \tag{5}$$

where $E_t=P_tC_t$, $E_t*=P_tC_t*$ are household expenditures at home and abroad. For logarithmic utility (1), the aggregate expenditures in both countries are constant over time, $E_t=E=(1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}Y_t$ and $E_t*=E*=(1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}Y_t*$.

2.2 Firms

Firms in each country are homogenous. Production function of a domestic firm at time period t is linear $y_t = x_t l_t$ where y_t is output, l_t labor input, x_t marginal labor productivity. Keeping the similar notation, $y_t{*} = x_t{*} l_t{*}$, for a foreign firm. Domestic and foreign firms face a fixed cost of production denoted f_t and $f_t{*}$.

Firms are monopolistic competitors: each sets a price markup θ over the unit production cost: $p_t = \theta w_t / x_t$, $p_t{*} = \theta w_t{*} / x_t{*}$ where w_t ($w_t{*}$) is domestic (foreign) wage. The net profit of domestic firms is equal to operating profit $\Pi_t = (p_t - w_t / x_t) y_t = \sigma^{-1} p_t y_t$ less investment in new technology and fixed cost: $\pi_t = \sigma^{-1} p_t y_t - \phi_t z_t - f_t$, where z_t denote the volume of investment, ϕ_t the domestic price of investment. Similarly, $\pi_t{*} = \Pi_t{*} - \phi_t{*} z_t{*} - f_t{*}$ where $\Pi_t{*} = \sigma^{-1} p_t{*} y_t{*}$.

The initial productivity of firms is x_0 or $x_0{*}$. They maximize the net present value of profits

$$v_0 = \sum_{t=1}^{\infty} \beta^t \pi_t \,,\ v_0{*} = \sum_{t=1}^{\infty} \beta^t \pi_t{*} \tag{6}$$

by selecting a sequence of investment z_t or $z_t{*}$ subject to a production function for technology improvement:

$$\Delta x_t = b_t \bar{x}_{t-1} z_t \,, \quad \Delta x_t{*} = b_t{*} \bar{x}_{t-1}{*} z_t{*} \tag{7}$$

and a non-negativity constraint:

$$z_t, z_t{*} \geq 0 \,. \tag{8}$$

Here $\Delta x_t = x_t - x_{t-1}$ ($\Delta x_t{*} = x_t{*} - x_{t-1}{*}$) is an increase of productivity at time period *t*, parameter $b_t (b_t{*})$ indicates effectiveness of investment in technology and $\bar{x}_{t-1}$ ($\bar{x}_{t-1}{*}$) is the average productivity in the domestic (foreign) economy at the beginning of period *t*. The latter factor refers to a national knowledge spillover: a higher average level of productivity, prevailing in the economy, makes technology improvement less costly for all local firms.[1] A firm does not affect the average level of productivity. Due to the symmetry of firms, $\bar{x}_{t-1} = x_{t-1}$, $\bar{x}_{t-1}* = x_{t-1}*$, and these equilibrium conditions are internalized by firms.[2]

Firms transfer fixed costs $f_t(f_t*)$ and investment expenditure $\phi_t z_t(\phi_t * z_t*)$ to local households that own claims on all profits at home. Households are supposed to be endowed, beyond labor, with relevant factors rewarded by fixed cost payments and investment expenses of firms. Parameters f_t, f_t* and ϕ_t, ϕ_t* are exogenous; the former can be treated as a lump-sum premium for establishing a new firm, and the latter as the marginal cost of investment in new technology.[3]

2.3 The number of firms

In each country the cost of entry is zero. The numbers of firms are endogenous and adjust at any period of time to satisfy the condition of zero net present value:

$$v_t = 0, \qquad v_t* = 0 \tag{9}$$

where $v_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t}\pi_\tau$, $v_t* = \sum_{\tau=t}^{\infty} \beta^{\tau-t}\pi_\tau*$. A new firm has free access to the "state of the art" technology of the past period, entailing the symmetry of firms in each economy. At any period the number of firms is determined first, then existing firms chose the volume of investment, and finally, given the new technology is installed, they make output-pricing

[1] The similar assumption is utilized in the Grossman and Helpman's model (1990, 1991, 1995), where productivity of research is increasing with accumulated knowledge indicated (in the case of local spillovers) by the number of prevailing local varieties.

[2] The assumption that representative firms and households internalize this symmetry as an equilibrium condition is used implicitly in the standard models of trade and/or growth with monopolistic competition, e.g. referred to in the previous footnote.

[3] Without loss of generality, the price of investment $\phi_t(\phi_t*)$ can be set to 1 because, as will be seen in what follows, it enters the equilibrium solution as the effective price $\phi_t / b_t(\phi_t * / b_t*)$

decisions and markets clear. According to such time sequencing, a firm does not affect the number of firms when deciding about investment and output.

2.4 Trading equilibrium

Trading equilibrium path is defined as a solution to the household problem (1)-(5) and the firm's problem (6)-(8) satisfying at each time period entry conditions (9), market-clearing conditions for the common market for goods:

$$y_t = c_{dt} + c_{it}*, \quad y_t* = c_{dt}* + c_{it}, \tag{10}$$

and market clearing conditions for the local labor markets

$$n_t l_t = 1, \quad n_t * l_t * = 1. \tag{11}$$

According to (11), labor in both countries is inelastically supplied to production. Market-clearing conditions (10) and the household budget constraints (5) imply the balance of payments: $n_t p_t y_t + n_t * p_t * y_t * = E + E*$ and equality of global saving and borrowing at any period of time.

To close the model we select a normalizing equation that determines the aggregate price level. A convenient way of normalization for the case of time-constant household expenditures is to set global expenditures to unity:

$$E + E* = 1, \tag{12}$$

with E and $E*$ representing the shares of countries in global spending (as in Grossman-Helpman' s model 1991, p. 160).

The aggregate household income is equal to the country's share in current global output, $Y_t = s_t$, $Y_t* = 1 - s_t$ where $s_t = n_t p_t y_t$, $1 - s_t = n_t * p_t * y_t *$. Aggregate expenditure is the permanent (weighted time-average) share of the country in global production, $E = (1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}s_t$, $E* = (1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}(1-s_t)$. If the current market share in period t exceeds the permanent one, the country is a net lender, otherwise it is a net borrower.

3. Equilibrium terms of trade

We characterize the trading equilibrium by focusing first on the interior solution of the firms' problem (6)-(8) implying positive investments in technology for both countries. According to the time sequencing within one period, a firm accounts for the direct effect of productivity shift on its revenue neglecting any influence of this shift on the firms' numbers. The latter are determined from entry conditions (9) implying zero net profits:

$$\pi_t = \sigma^{-1} r_t - \phi_t z_t - f_t = 0, \qquad \pi_t * = \sigma^{-1} r_t * - \phi_t * z_t * - f_t * = 0. \qquad (13)$$

where $r_t = p_t y_t$, $r_t * = p_t * y_t *$ are the revenues of firms.

Let $h_t = r_t * / r_t = p_t * (p_t *)^{-\sigma} / p_t (p_t)^{-\sigma} = \left(w_t x_t * / w_t * x_t\right)^{\sigma-1}$ denote the equilibrium relative revenue. This is an increasing power function of price of domestic exports relative to imports $p_t / p_t *$ indicating the advantage of foreign firms in trade. For the sake of convenience the relative revenue h_t, instead of the relative price $p_t / p_t *$, will be called in what follows the terms of trade.

3.1. Revenues and numbers of firms

Combining market-clearing conditions (10), household budget constraints per period (5) and normalizing equation (12), and taking into account that the symmetry of local firms is internalized in their decisions, yields the revenues:

$$r_t = (n_t + n_t * h_t)^{-1}, \quad r_t * = h_t (n_t + n_t * h_t)^{-1} \qquad (14)$$

Given the numbers of firms, the domestic firm's revenue is decreasing in h_t and increasing in x_t. It is strictly concave in x_t for $1 < \sigma \le 2$, and S-shaped for $\sigma > 2$ (figure 1).

Figure 1. Revenue as a function of productivity

Lemma 1. Investments of domestic and foreign firms are

$$z_t = \frac{(1-s_t)f_t - \mu\varphi_t}{\phi_t(s_t+\eta)}, \; z_t^* = \frac{s_t f_t^* - \mu\varphi_t^*}{\phi_t^*(\mu - s_t)} \tag{15}$$

where $\varphi_t = \phi_t / b_t$, $\varphi_t^* = \phi_t^* / b_t^*$ *is the effective price of investment,* $\eta = (2-\sigma)/(\sigma-1)$.

Proposition 1. The equilibrium revenues of firms are

$$r_t(h_t) = (\psi_t^* + \psi_t h_t)/\omega h_t, \qquad r_t^*(h_t) = (\psi_t^* + \psi_t h_t)/\omega. \tag{16}$$

where $\psi_t = f_t - \varphi_t$, $\psi_t^* = f_t^* - \varphi_t^*$. *The equilibrium numbers of firms are*

$$n_t(h_t) = \frac{\omega s_t h_t}{\psi_t^* + \psi_t h_t} \quad n_t^*(h_t) = \frac{\omega(1-s_t)}{\psi_t^* + \psi_t h_t}, \tag{17}$$

where $\omega = (3-\sigma)/\sigma$.

The effective prices of investment φ_t and φ_t^* have a meaning of the period *t* cost of new technology under incremental productivity growth. Indeed, one can check from the proof of Lemma 1 that the marginal cost of productivity growth in period *t* is φ_t / x_{t-1}. Considering incremental increase and multiplying this marginal cost by x_t close to x_{t-1} yields φ_t. Therefore φ_t and φ_t^* indicate productivities in period *t* evaluated at marginal costs of incremental productivity growth. Parameters ψ_t and ψ_t^* measure the *net fixed cost* of production in period *t* that is the fixed cost of production less the cost of current productivity.[4]

[4] Permitting fixed non-zero and time-varying entry costs q_t and q_t^* that modify equations (9) to $v_t = q_t$ and $v_t^* = q_t^*$ would not alter the model inferences. In this case the net fixed costs of production include the alternative costs of a new firm creation in period *t*: $\psi_t = f_t - \varphi_t + q_t - \beta q_{t+1}$ and $\psi_t^* = f_t^* - \varphi_t^* + q_t^* - \beta q_{t+1}^*$.

In equilibrium, the net fixed cost coincide with *a productivity surplus* $\Pi_t - \frac{\partial \Pi_t}{\partial x_t} x_t$ or $\Pi_t * - \frac{\partial \Pi_t *}{\partial x_t *} x_t *$, obtained by a domestic or foreign firm, respectively. This is a consequence of the first-order condition for the firm's problem, as given by (A1) in the proof of lemma 1. It can be rewritten for a domestic firm as $\frac{\partial \Pi_t}{\partial x_t} x_t - \phi_t z_t = \varphi_t$. Subtracting this from the zero-profit condition (13) implies that the net fixed cost is equal to the productivity surplus of the domestic firm:

$$\psi_t = \Pi_t - \frac{\partial \Pi_t}{\partial x_t} x_t .$$

For $1 < \sigma \leq 2$ the operating profit Π_t is strictly concave in x_t. For $\sigma > 2$ this is an S-shaped function of productivity, but in equilibrium x_t belongs to the zone of decreasing returns. Therefore ψ_t must be positive in any case, and $\psi_t * = \Pi_t * - \frac{\partial \Pi_t *}{\partial x_t *} > 0$ as well. Consequently, non-positive net fixed costs ψ_t or $\psi_t *$ are ruled out for the second-order conditions of the firms' problem to hold.

The sum of foreign and domestic productivity surpluses weighted by inverse of the terms of trade can be defined as *a combined productivity surplus*. According to proposition 1, the revenues of firms are proportional to this surplus: $\psi_t * / h_t + \psi_t$ at home and $\psi_t * + \psi_t h_t$ abroad. From (17), the higher are the terms of trade, the lower is the weight of the foreign productivity surplus and the revenue obtained by a domestic firm.

The firm's revenue is increasing with the elasticity of substitution σ. Intuitively, if this elasticity is low, firms specialize in strongly differentiated goods and are able to sustain tough competition of the large number of small-sized firms. If σ is close to the limit case 3, the international market for goods is dominated by a small number of large firms. Strictly speaking, this case is falling out of the framework of monopolistic competition.[5] The permissible range for the elasticity of substitution, $\sigma \in (1,3)$, looks very restrictive on the first glance. Empirical estimates of σ for specific industries normally vary in the range 2 – 10 and higher for some cases (Feenstra 1994, Lai and Treffler 1999, Obstfeld, Rogoff, 2000). But parameter σ in our model is the intratemporal elasticity of substitution for the national economy that can be regarded as a mixture of industries. The inter-industry elasticity of substitution is normally quite low (far below 1 for complementary ones), and estimates of σ on the national base should be well below the specific industry estimates, as will be seen from the discussion that follows.

3.2. Equilibrium market share

The proof of proposition 1 yields equilibrium market shares:

$$s_t(h_t) = \frac{\mu\psi_t h_t - \eta\psi_t{}^*}{\psi_t{}^* + \psi_t h_t}, \qquad 1 - s_t(h_t) = \frac{\mu\psi_t{}^* - \eta\psi_t h_t}{\psi_t{}^* + \psi_t h_t} \tag{18}$$

The global demand is divided between the countries according to the rule relating the linear combination of productivity surpluses $\mu\psi_t h_t - \eta\psi_t{}^*{}_t$ or $\mu\psi_t{}^* - \eta\psi_t h_t$ to the combined productivity surplus. The case $\sigma = 2$ is a benchmark: the market share is exactly equal to the country's share in the combined productivity surplus, $s_t(h_t) = \psi_t h_t / (\psi_t{}^* + \psi_t h_t)$.

Market shares (18) define a trading equilibrium only if $s_t \in (0,1)$. For $\sigma > 2$ this condition is fulfilled, and for $1 < \sigma \le 2$ the market shares are in the unit interval if

[5] Trading equilibrium does not exist for $\sigma \ge 3$ because the firms' revenues, as defined by (16), are indefinite for $\sigma = 3$ and negative for $\sigma > 3$. The failure of equilibrium to exist for higher elasticity of substitution is clarified by appealing to investment equations (15). One of the following pairs of conditions is incompatible for $\sigma \ge 3$: $z_t, \psi_t > 0$ or $z_t{}^*, \psi_t{}^* > 0$. Indeed, let $s_t \ge 1/2$. Then the numerator in equation (15) for $z_t{}^*$ is positive, because $(\sigma - 1)s_t f_t{}^* - \varphi_t{}^* > f_t{}^* - \varphi_t{}^* = \psi_t{}^* > 0$. The denominator in this equation must be positive for $z_t{}^* > 0$, implying $s_t < \mu < 1/2$. Similarly, by letting $s_t < 1/2$ one can show that either z_t or ψ_t is non-positive for $\sigma \ge 3$. Consequently, positive investment is inconsistent with the second-order condition for the firms' problem in this case, and therefore we restrict σ from being equal or above 3.

$$\frac{\psi_t *}{\psi_t}(2-\sigma) < h_t < \frac{\psi_t *}{\psi_t}(2-\sigma)^{-1}. \qquad (19)$$

The interval of admissible h_t is thus getting very narrow for σ close to 1 and indefinitely widens for σ close to 2.

Equilibrium market share of domestic firms is an increasing function of the terms-of-trade h_t because, from (18),

$$s_t'(h_t) = \frac{(\mu+\eta)\psi_t\psi_t *}{(\psi_t * + \psi_t h_t)^2} = \frac{(3-\sigma)\psi_t\psi_t *}{(\sigma-1)(\psi_t * + \psi_t h_t)^2} > 0. \qquad (20)$$

The terms of trade affect the market share in a twofold way. The short-term direct effect is negative since, holding constant the numbers of firms, the market share is decreasing in the terms of trade. A longer-term indirect effect is determined by changes in the equilibrium numbers of firms. An increase of the terms of trade encourages entry of domestic firms and discourages entry of foreign firms: $n_t'(h_t) > 0$, $n_t *'(h_t) < 0$. The indirect effect of the terms of trade on the market share is positive and outweighs the direct relative price effect.

Equation (20) clearly demonstrates counteraction of these effects in the market share determination. Their relative power depends on the degree of differentiation of goods (or specialization of firms). The market share is responsive to changes in the terms of trade for strongly differentiated goods ($1 < \sigma < 2$) and is less responsive otherwise ($2 < \sigma < 3$). In the former case, the indirect effect is profound since a change of h_t shifts the numbers of firms dramatically if σ is low, and the direct effect is weak since the relative demand is low-elastic. The indirect effect, then, is strongly dominating, and the market share is responsive to shifts in h_t. In the latter case, the indirect effect is still dominating but the market share is only slightly affected by the terms of trade if σ is high.

4. The relative productivity and the terms of trade

We mentioned above two kinds of causal links between the relative productivity and the terms of trade emerging in our model. The static link is conditioned by interaction between relative productivity and the concurrent terms-of-trade through the relative wage determination. The dynamic link stems from the effect of the terms of trade on the investment decisions of firms. Interaction of these links defines a unique trading equilibrium at each period of time and generates productivity dynamics.

4.1 The static link between productivity and terms of trade

An equilibrium *static link* between concurrent terms-of-trade and relative productivity $\xi_t = x_t / x_t *$ is given by the relationship: $\xi_t = (w_t / w_t *) h_t^{-\mu}$. The relative wage is determined from the labor market-clearing conditions (11) implying that absolute wages are proportional to the market shares of countries[6]:

$$w_t = s_t(h_t) / \theta \tag{21}$$

$$w_t * = (1 - s_t(h_t)) / \theta \tag{22}$$

The absolute wages are defined by (21), (22) implicitly, because the relative wage $w_t / w_t *$ enters the right-hand sides. Dividing both sides of (21) on (22) yields a static terms-of-trade equation $\xi_t = \dfrac{s_t(h_t)}{1 - s_t(h_t)} h_t^{-\mu}$ or, due to (18):

$$\xi_t = \frac{\psi_t h_t - (2-\sigma)\psi_t *}{\psi_t * - (2-\sigma)\psi_t h_t} h_t^{-\mu} \equiv \xi_t(h_t) \, . \tag{23}$$

Proposition 2. The static link is positive, $\xi_t'(h_t) > 0$ *, for* $1 < \sigma < 2$ *and negative,* $\xi_t'(h_t) < 0$ *, for* $2 < \sigma < 3$.

[6] Since $y_t n_t = s_t / p_t$, the labor market-clearing equation (11) transforms to: $s_t / p_t x_t = 1$. Once $p_t = \theta w / x_t$, this is equivalent to (21). Equation (22) is obtained similarly.

A direct effect of ξ_t on h_t is negative, but since the relative wage depends on the terms of trade, the sign of the static link is ambiguous. An increase of relative productivity ξ_t is counterbalanced by a rise in relative wage implying an expansion of domestic market share and, as a result, a terms-of-trade improvement. For the case $1 < \sigma < 2$ this positive indirect effect of relative wage determination dominates the negative direct effect of ξ_t on h_t. For σ close to 1 the function $\xi_t(h_t)$ is nearly vertical in the admissible neighborhood of the point $h_t = \psi_t * / \psi_t$ satisfying (19). For σ close to 2, $\xi_t(h_t)$ is nearly horizontal. The static link vanishes in the benchmark case, $\sigma = 2$, when the relative productivity is constant and equal to the ratio of productivity surpluses $\xi_t = \psi_t / \psi_t *$.

For higher elasticity of substitution the static link is negative. The direct effect of productivity gain on the terms of trade dominates the indirect effect of relative wage increase. The number of firms grows slowly with h_t, and the terms of trade weakly affects the market share and the relative wage. Due to this, a gain in relative productivity causes a reduction of relative wage, domestic market share and, as a result, the terms of trade. For σ close to 3 the static link is given approximately by $\xi_t(h_t) = h_t^{-1/2}$.

The function $\xi_t(h_t)$ is represented in figure 2. The vertical asymptote $h_t = \psi_t * / \psi_t (2 - \sigma) \equiv \overline{h}_t$ corresponds to the upper bound of the unit interval of s_t in the case of positive static link. The lower bound in this case is $\underline{h}_t \equiv (2 - \sigma)\psi_t * / \psi_t$.

Figure 2. The static link between ξ_t and h_t.

4.2. The dynamic link between productivity and terms of trade

The terms-of-trade effect on current productivity growth is negative: the equilibrium growth rates are[7]

$$\frac{\Delta x_t}{x_{t-1}} = \varphi_t^{-1}\left(\frac{\psi_t * + \psi_t h_t}{(3-\sigma)h_t} - f_t\right),\ \frac{\Delta x_t *}{x_{t-1} *} = (\varphi_t *)^{-1}\left(\frac{\psi_t * + \psi_t h_t}{(3-\sigma)} - f_t *\right). \qquad (24)$$

These rates are proportional to the weighted difference of the combined productivity surplus and the fixed cost of production at home. The relative productivity growth of the domestic economy is decreasing in h_t:

$$\frac{\xi_t}{\xi_{t-1}} = \frac{1+\varphi_t^{-1}\left(\frac{\psi_t * + \psi_t h_t}{(3-\sigma)h_t} - f_t\right)}{1+(\varphi_t *)^{-1}\left(\frac{\psi_t * + \psi_t h_t}{(3-\sigma)} - f_t *\right)} = \frac{\varphi_t *[(3-\sigma)(\varphi_t - f_t) + \psi_t + \psi_t * / h_t]}{\varphi_t[(3-\sigma)(\varphi_t * - f_t *) + \psi_t * + \psi_t h_t]}$$

Combining this equation with (23) and rearranging terms implies $\xi_t / \xi_{t-1} = (\varphi_t * / \varphi_t)(\xi_t h_t^{1+\mu})^{-1}$ or, since $1+\mu = \theta$:

$$\xi_t (h_t)^2 = \xi_{t-1} h_t^{-\theta} / \vartheta_t \qquad (25)$$

where $\vartheta_t = \varphi_t / \varphi_t *$ is the relative cost of technology. According to (25), the relative productivity is proportional to the geometric average of lagged relative productivity and current relative demand $h_t^{-\theta} = (p_t / p_t *)^{-\sigma}$. Let $g_t(h_t) = 1/(\xi_t(h_t) h_t^{\theta} \vartheta_t)$ denote growth rate of relative productivity, ξ_t / ξ_{t-1}, as a function of the terms of trade. Then the solution of (25) $h_t = h_t(\xi_{t-1}) = \left(\xi_t(h_t) / g_t(h_t)\right)^{-1}$ indicates *a dynamic link* between the lagged relative productivity and the terms-of-trade.

Proposition 3. The dynamic link is positive, $h_t'(\xi_{t-1}) > 0$.

The dynamic link is positive because for $1 < \sigma < 2$ the left-hand side of (25) is increasing in h_t and the right-hand side is decreasing. For $2 < \sigma < 3$ the left-hand side is

---

7 The firms' operating profits are $\Pi_t = \sigma^{-1} r_t(h_t) = (\psi_t * + \psi_t h_t)/(3-\sigma)h_t$ and $\Pi_t * = \sigma^{-1} r_t *(h_t) = (\psi_t + \psi_t * h_t)/(3-\sigma)$. Substituting these into zero-profit conditions (13) and

decreasing slower than the right-hand side, and again we have $h_t'(\xi_{t-1}) > 0$. The negative dependence of relative productivity growth on the terms of trade implies the positive dependence between h_t and ξ_{t-1}, which is either reinforced by the positive static link between ξ_t and h_t, or dominates this link, if it is negative.

The dynamic link emerges because of the interaction of relative wage determination with investment decisions by firms. On the one hand, a lagged productivity gain brings into action the local knowledge spillover that allows a firm at home to reduce investment needed to reach a given level of current productivity. Productivity growth is therefore financed with a lower amount of current operating profits. On the other hand, the lagged productivity gain enforces increase of the relative number of domestic firms in the current period that, in turn, fosters relative wage growth outstripping current productivity growth. A terms-of-trade improvement is required for balancing these effects, and the resulting causal link is $h_t(\xi_{t-1})$.

The strength of this link, as indicated by the slope of $h_t(\xi_{t-1})$, depends crucially on the intratemporal elasticity of substitution. Indeed, if $1 < \sigma < 2$ and both $\xi_t(h_t)$ and $1/g_t(h_t)$ are increasing in h_t, then $\xi_{t-1} = \xi_t(h_t)/g_t(h_t)$ is a steep function of h_t. This ratio flattens for $2 < \sigma < 3$, because $\xi_t'(h_t) < 0$. Hence, the slope of $h_t(\xi_{t-1}) = \left(\xi_t(h_t)/g_t(h_t)\right)^{-1}$ is low in the former case and high in the latter. In other words, the dynamic link is weak for low elasticity of substitution and strong otherwise. Intuitively, the market share is responsive to current shifts in the terms of trade if σ is low, as seen from the above discussion of equation (20). The relative wage is responsive to these shifts as well, as clear from (21)-(22). Then the relative wage adjustment required to meet the current productivity growth does not cause significant shifts in the terms of trade. By contrast, the required change of the terms of

---

accounting for (7) yields (24).

trade may be substantial for high σ, implying a strong dynamic effect of the lagged relative productivity on the terms of trade.

The following examples are illustrative and shown in figure 3. For σ close to 1 the terms of trade is approximately constant, $h_t = \psi_t * / \psi_t$. For $\sigma = 2$ the relative productivity is constant, $\xi_t = \psi_t / \psi_t *$ and, from (25), $h_t(\xi_{t-1}) = \xi_{t-1}^{1/2}(\psi_t * / \psi_t)\vartheta_t^{-1/2}$. For σ near 3 the static link is roughly $\xi_t(h_t) = h_t^{-1/2}$, and (25) implies that $h_t(\xi_{t-1}) = \xi_{t-1}^2 \vartheta_t^{-2}$. The shape of the dynamic link thus varies from nearly constant (for σ close to 1) to nearly quadratic (for σ close to 3).

Figure 3. The dynamic link between h_t and ξ_{t-1}.

4.3 Existence of trading equilibrium

Figure 4(a,b) depicts equation (25). The left-hand side is drawn with continuous line and the right-hand side with dotted line. The unique intersection point $h_t = h_t(\xi_{t-1})$ corresponds to the solution of (25) that constitutes a unique trading equilibrium for period *t* if it satisfies (19) requiring that $0 < s_t < 1$, and conditions of positive growth: $\Delta x_t > 0$, $\Delta x_t * > 0$. Condition (19) is fulfilled for $2 \leq \sigma \leq 3$, and it is also fulfilled for $1 < \sigma < 2$ in equilibrium, because $h_t(\xi_{t-1})$ is located between the vertical asymptote $h_t = \overline{h}_t$ and the lower bound $h_t = \underline{h}_t$, as seen from figure 4(a). Conditions of two-country positive growth are essential, because the trading equilibrium analyzed so far is based on the interior solutions of the firms' problems for both countries. These conditions are

$$\frac{(3-\sigma)f_t * - \psi_t *}{\psi_t} < h_t(\xi_{t-1}) < \frac{\psi_t *}{(3-\sigma)f_t - \psi_t} . \quad (26)$$

They are more restrictive than (19), and fulfilled if either technology costs φ_t, φ_t* are small or σ is high[8].

Figure 4. Trading equilibrium

Thus, the trading equilibrium in period *t* exists and unique, provided that growth is positive for $h_t = h_t(\xi_{t-1})$ in both countries. A time path for the relative productivity and the terms of trade is determined by the productivity growth equation (25) represented as

$$\xi_t^2 = \xi_{t-1} h_t(\xi_{t-1})^{-\theta} / \vartheta_t . \tag{27}$$

The equivalent difference equation is $\xi_t = \xi_t(h_t(\xi_{t-1}))$ meaning the superposition of static and dynamic links. Let (26) be fulfilled at any period of time $t \geq 1$. Then, given the initial relative productivity ξ_0 and a sequence of the model parameters $\varphi_t, \varphi_t*, f_t, f_t*(t \geq 1)$, equation (27) determines a unique trading equilibrium path.

However, one of the positive growth conditions (26) may be binding for a large productivity gap in period *t*-1. Let the domestic economy be backward in this period implying that ξ_{t-1} is quite low. Then $h_t(\xi_{t-1})$ is also quite low, violating the left inequality in (26). Because of the productivity gap in the previous period, the terms of trade $1/h_t$ proves to be too high for foreign firms to invest in productivity. In this situation the trading equilibrium exists only if foreign firms stop investment for a while, enabling domestic firms to reach a certain level of relative productivity satisfying the left inequality in (26). This finite-time piece of growth path is based on the asymmetric trading equilibrium with a corner solution examined in what follows.

[8] One can rewrite (26) as $\underline{h}_t + (3-\sigma)\dfrac{\varphi_t*}{\psi_t} < h_t(\xi_{t-1}) < \bar{h}_t\left(1 + (3-\sigma)\dfrac{\varphi_t}{\psi_t*}\bar{h}_t\right)^{-1}$.

5. Trade and mean-reversion of relative productivity

The key dynamic property of our model is that trade in goods implies conditional convergence of productivities. To demonstrate this property, consider relative productivity dynamics for the interior trading equilibrium path.

5.1. Productivity races.

Let both conditions of positive investment (26) be fulfilled for both countries and at any time, and consider the equation of relative productivity growth (27). Dynamic properties of (27) are revealed for time-constant model parameters $\varphi_t, \varphi_t*, f_t, f_t*$. The autonomous difference equation is $\xi_t = \xi(h(\xi_{t-1}))$ with $\xi(h_t)$ and $h(\xi_{t-1})$ indicating time-invariant static and dynamic links. The steady state terms of trade $\hat{h}$ solves equation $g(h) = 1$ which is quadratic with one positive root. The steady state relative productivity is $\hat{\xi} = \xi(\hat{h})$, and for the symmetric model parameters $\hat{\xi} = \hat{h} = 1$.

The steady state is globally stable for values of σ under consideration. In the limit case $\sigma = 1$ dynamic equation (27) is defined only for $\psi = \psi*$ and formally written as $\xi_t = (\xi_{t-1} / \vartheta)^{1/2}$. There is continuum of periodic trajectories in the opposite limit case $\sigma = 3$ implying formally $\xi_t = \vartheta / \xi_{t-1}$. The productivity gap is oscillating and sign-alternating, and convergence is slow for σ close to 3. The productivity gap gradually vanishes for all σ in the permissible interval (1, 3). Global convergence to the steady state is monotonous for $1 < \sigma < 2$ and cyclical for $2 < \sigma < 3$. In the benchmark case $\sigma = 2$ the steady state $\hat{\xi} = \psi / \psi*$ is reached in one period of time, and convergence is rapid for σ close to 2 (this should not be taken too literally, because a time period is sufficiently long in our model). Figure 5 depicts three kinds of global transition: a) monotonous convergence ($1 < \sigma < 2$), b) one-period transition ($\sigma = 2$), and c) oscillating convergence ($2 < \sigma < 3$).

Figure 5. Convergence of relative productivity

The model thus exhibits mean reversion and global stability of relative productivity dynamics. Convergence under constant parameters implies conditional convergence for the basic model with time-varying parameters. This property is a consequence of a damping mechanism of international trade. On the one hand, the terms of trade of the lagged economy is relatively low and fostering growth, while growth of the leading economy is slowing down by the same reason. The relative productivity of the less advanced country increases, squeezing the initial productivity gap. On the other hand, the pace of relative productivity growth is slowing down in time, as well as the pace of the terms-of-trade growth of this country. The damping effect arises because the positive dynamic link affects productivity growth rate, which is decreasing in the current terms of trade. Importantly, convergence is revealed for the growth model without capital and diminishing returns to investment.

The intratemporal elasticity of substitution affects the patterns of convergence, because it determines the strength of the dynamic link. As we have seen, the slope of $h_t(\xi_{t-1})$ is not large for low σ (since the indirect terms-of-trade effect on the market share is significant), and the lagged productivity gap does not cause considerable terms-of-trade variation. As a result, the induced shifts in current relative productivity are not dramatic. By the contrast, the indirect terms-of-trade effect on the market share is weak if σ is above 2. Due to this, the terms of trade are quite responsive, and the dynamic link is strong enough to generate the pattern of alternating technological leadership.

5.2. Interconnection of trade and growth: a discussion

Let us summarize and discuss the above results. Our model is designed to examine causal effects of productivity gains on the terms of trade, on the one hand, and converse terms-of-trade effects on growth, on the other hand. The former are embodied in the dynamic link between the past period relative productivity and the present period terms of trade. This link is resulting from interaction of three kinds of effects: a) the negative direct effect of productivity gain on concurrent terms of trade; b) the ambiguous static dependence between these variables involving determination of market share and relative wage; c) the terms-of-trade improvement required for balancing the relative wage growth and the shift of investment by firms in response to the lagged gain in relative productivity. The resulting dynamic link is positive and weak (strong) for low (high) elasticity of substitution.

This link essentially determines the patterns of converse effects of trade on long-run productivity growth in our model. There is no growth under autarky regarded as a special case of the model with trade. Trade induces growth due to the revenue-expanding motive of firms exploiting limited monopoly power and seeking to shift the terms of trade through investment in technology.[9] Besides that, trade ensures mean-reversion of relative productivity under quite general provisions. Dynamics of this variable are globally stable irrespective of the size of initial productivity gap and structural characteristics of the economies.

The relative productivity is converging because the terms of trade determine current competitive environment in trading countries and investment opportunities of firms. This is demonstrated by referring to the low of motion for relative productivity (27). It looks quite simple and intuitively appealing, but can still further be simplified. Taking logarithms on

[9] This motive is absent in autarky, and both economies stagnate if they do not trade. Conditions of autarky equilibrium are obtained from our model with trade by setting n_t* to 0 for the domestic economy and n_t to 0 for the foreign economy. In particular, the aggregate consumption indices for the autarky case are $C_t = \left(\int_0^{n_t} c_{dt}(j)^{1/\theta} dj \right)^{\theta}$, $C_t* = \left(\int_0^{n_t*} c_{dt}*(j)^{1/\theta} dj \right)^{\theta}$. Households and firms solve their problems (1)-(5) and (6)-(8) under free entry conditions (9) and market-clearing conditions for labor (11) and goods: $y_t = c_{dt}$, $y_t* = c_{dt}*$. Two normalizing equations determine the level of aggregate expenditure for each country. Letting without loss of generality $E = E* = 1/2$ yields $r_t = 1/2n_t$ and $r_t* = 1/2n_t*$. Due to the time sequencing within one period, a firm does not affect the number of firms and is unable to expand its revenue by

both sides of (27) and accounting for equality of relative demand and supply, $h_t^{-\theta} = y_t / y_t *$, and letting $\vartheta_t = 1$, we have: $\tilde{\xi}_t = 0.5(\tilde{\xi}_{t-1} + \tilde{y}_t - \tilde{y}_t *)$, where tilde means log. Due to the labor market-clearing conditions (11), this low of motion is rewritten as

$$\Delta\tilde{\xi}_t = \tilde{n}_t * -\tilde{n}_t$$

The relative productivity growth is inverse of the relative number of firms, which is increasing in the terms of trade and (through the dynamic link) in the lagged relative productivity. On the other hand, the low terms-of-trade faced by the less advanced economy exerts a positive valuation effect on the domestic firm's revenue by raising the weight of foreign productivity surplus (according to (16), the equilibrium revenue is proportional to the combined productivity surplus weighted by inverse of the terms of trade). As a result, this economy is represented by the relatively small number of relatively large-sized firms. Trade in goods, thereby, not only determines relative prices, but governs competitive pressures on firms by mitigating these for the backward economy and toughening for the leading economy. The positive dynamic link between productivity and terms of trade is growth-conducive in the former and detrimental in the latter. Such converse causality is consistent with the Shumpeterian view that tough competition is harmful for innovation-based growth (Aghion, Howitt 1998).

Thus, the mechanism of convergence rests on the model property that the leading nation exports relatively large number of varieties at higher price level. This property is consistent with cross-country findings by Hummels and Klenow (2005) relating extensive and intensive export margins to GDP per worker and the country size. These authors show that the extensive margin plays a prominent role for rich countries (op. cit., p. 712, table 2) exporting higher quantities per worker without lowering prices of their varieties (op. cit., p.

---

investment in productivity. As a result, the economies stagnate under autarky. This is an extremely stylized formalization of the old idea that trade drives growth, which is not a subject of our paper.

713, table 3). Our model also predicts, similarly to the trade models with quality differentiation (e.g. Flam, Helpman 1987, Grossman, Helpman, 1991) and consistently with the findings of Hummels and Klenow, that the economy with higher GDP per worker can export more quantities of each variety at higher prices.[10] This is seen from the dynamic equation (27) represented in log-additive form: $\tilde{\xi}_t = 0.5(\tilde{\xi}_{t-1} + \tilde{y}_t - \tilde{y}_t^*)$ and linking a higher relative productivity of the nation with a higher firm-level output.

Our model exhibits monotonous convergence for the elasticity of substitution below the benchmark $\sigma = 2$ and oscillating convergence for σ above this benchmark. The former case supports Barro's conclusion that terms-of-trade improvement should stimulate long-run growth. If the initial productivity of domestic economy is relatively low, then the terms-of-trade in period 1 (after opening trade) are also low. The positive dynamic link causes fostering of relative productivity growth of the lagged country at the beginning of transition path and subsequent terms-of-trade improvement. These two processes complement and dampen each other over time, consistently with the Barro's (1999) evidence.

The dynamic link is strong enough for $\sigma > 2$ to generate alternating leadership across countries. In this case the static link is negative and the mapping (27), presented as the superposition of these links, $\xi_t = \xi_t(h_t(\xi_{t-1}))$, is downward sloping implying oscillating convergence. The mechanism of overtaking of the advanced economy is similar to one examined in the models of trade and growth with alternating leadership. For instance, the pioneering paper by Brezis et al. (1993) demonstrates overtaking in the presence of a learning-by-doing externality resulting in superior experience of the leading nation in the existing technology. The leadership is punished by the relative wage growth, precluding

[10] Hummels and Klenow (2005) compare several workhorse models of trade, basing on Feenstra's (1994) methodology. They demonstrate, for example, that the Armington (1969) model with product differentiation across countries and no extensive margins is omitting more than half of exports of larger economies and predicting, contrary to data, that richer countries will export higher quantities of each variety at lower prices. On the other hand, the Krugman models of monopolistic competition and endogenous number of firms (1979,

switch to a new technology by the leading economy and stimulating this for the backward economy (op. cit., p. 1217). In our model the advanced economy loses its position because of the substantial terms-of-trade improvement caused by the relative wage increase in response to the lagged productivity gain.

Being consistent with intuition, this pattern of growth does not fit well the post-war evidence on productivity dynamics across countries. Cases of overtaking occur sporadically, but there is no regularity showing cycles of alternating leadership among national economies. Leadership reversals are observed on the very long historical horizons and explained with reference to a wide array of socio-economic factors in addition to technological competition (e.g., Kindleberger 1996, Artige et al. 2003). Thus, on the one hand, evidence on post-war cross-country growth is supporting monotonous (conditional) convergence of productivities rather than alternating technological leapfrogging across countries. On the other hand, evidence from the international business cycle research suggests the case of low Armington elasticity of substitution for national economy as a whole.[11] For example, parameter σ is set equal to 1.5 by Backus et al. (1995), with reference on a large number of empirical studies (op. cit., p. 347). Our model predicts, consistently with these observations, monotonous convergence of relative productivity just for the case of low elasticity of substitution, $1 < \sigma < 2$.

It is interesting to relate this inference to the prediction by Acemoglu and Ventura (2002) concerning the dependence between terms of trade and GDP growth and obtained for the abovementioned model of global income distribution. The terms of trade in their model are unambiguously negatively related to a country relative income, as implied by the static trade balance equation (number 12 in their paper) featuring direct supply-demand causality

---

1980, 1981) feature a prominent extensive margin for rich economies, but predicts no effect of GDP per worker on unit price of exports (Hummels, Klenow 2005, p. 708).

[11] In our view, the Armington elasticity is a proper measure of substitubility across all varieties produced by the national economy, unlike estimates for specific industries or categories of goods.

and underlying the mechanism of income convergence. This prediction is supported by a cross-country regression of terms-of-trade growth on GDP growth and a set of control variables. The implied price elasticity of export is $\sigma = 2.6$ with notation of our paper (op. cit., p. 673). This estimate corresponds to the case of negative static link in our model and would mean a strong terms-of-trade effect on growth, rendering alternation of leadership. Anyway, our results do not contradict to the finding by Acemoglu and Ventura (2002), because they examine the effect of pure capital accumulation net off technology improvement, which positively affects the terms of trade in their model.

Hummels and Klenow (2005) emphasize the closeness of the Acemoglu and Ventura (2002) and Armington (1969) models in their prediction that richer countries will export higher quantities of each variety at a lower price. The negative terms-of-trade effect prevents per capita incomes from diverging across countries and is critical for stability of world income distribution obtained by Acemoglu and Ventura (2002). Pointing out the discrepancy between this key property of their model and cross-country evidence on extensive margins, Hummels and Klenow (2005) make a conclusion that “diminishing returns and technology diffusion may be needed to ensure a stationary world income distribution” (op. cit., p. 713). As our results on convergence of relative productivity demonstrate, this is by no means the solely possible strategy of research.[12]

6. Waiting for the backward economy

As has been mentioned already, the interior trading equilibrium does not exist for a large productivity gap because one of the positive growth conditions (26) is violated. Let the domestic economy be backward in period *t*-1 to such an extent that ξ_{t-1} is too low for

[12] Commenting on the failure of the Krugman-style models with product differentiation by firms to match the positive relationship between prices and exporter income per worker, Hummels and Klenow (2005) suggest “modifying these models to include quality differentiation” (op. cit., p. 718). Most likely, this is a promising way to capture empirical regularities.

$h_t(\xi_{t-1})$ to satisfy the left inequality in (26). Foreign firms then face the binding non-negativity constraint (8) and do not invest, thereby letting domestic firms to reduce the gap. To deal with this situation, we consider an asymmetric trading equilibrium with a corner solution for foreign firms and an interior one for domestic firms.

Proposition 1a. In asymmetric trading equilibrium the revenues of firms are

$$r_t(h_t) = \sigma f_t * / h_t, \qquad r_t * (h_t) = \sigma f_t * \tag{16a}$$

and the numbers of firms are

$$n_t(h_t) = s_t h_t / \sigma f_t * \quad n_t * (h_t) = (1 - s_t) / \sigma f_t * . \tag{17a}$$

As above, combining the first-order condition with the zero-profit condition for domestic firms (13) yields the equality of the net fixed cost and the productivity surplus: $\psi_t = \Pi_t - \frac{\partial \Pi_t}{\partial x_t} x_t > 0$. The firm's operating profit is $\Pi_t = \sigma^{-1} r_t(h_t) = f_t * / h_t$ at home and $\Pi_t * = \sigma^{-1} r * (h_t) = f_t *$ abroad. The latter equation follows directly from the zero-profit condition (13), since foreign firms do not invest. The firm's revenue and operating profit are proportional to the foreign fixed cost, not the combined net fixed cost, as is the case for the interior trading equilibrium.

The terms of trade are defined through the relative wage equation $\xi_t(h_t) = \frac{s_t(h_t)}{1 - s_t(h_t)} h_t^{-\mu}$. Equilibrium market share is obtained in the proof of proposition 1a:

$$s_t(h_t) = -\eta + \mu \psi_t h_t / f_t * , \tag{18a}$$

and, consequently, the static link is

$$\xi_t(h_t) = \frac{\psi_t h_t - (2 - \sigma) f_t *}{f_t * - \psi_t h_t} h_t^{-\mu} . \tag{23a}$$

Proposition 2a. The static link is positive, $\xi_t'(h_t) > 0$ *, if 1 <* $\sigma \leq 2$*.*

The two cases of the static link between ξ_t and h_t are illustrated in figure 6(a,b), where function $\xi_t(h_t)$ is drawn with continuous curve. The point $\overline{h}_t = f_t{*}/\psi_t$ is the upper bound of h_t constraining the domestic market share from above 1. Figure 6 (a) shows the case of positive link for $1 < \sigma \le 2$, and figure 6 (b) demonstrates an U-shaped link for $2 < \sigma < 3$: $\xi_t(h_t)$ tends to + ∞, as h_t approaches 0 or $\overline{h}_t$.

Figure 6. Asymmetric trading equilibrium.

The zero-profit condition (13) implies the growth rate of relative productivity:

$$\frac{\Delta\xi_t}{\xi_{t-1}} = \frac{\Delta x_t}{x_{t-1}} = \varphi_t^{-1}\left(\frac{f_t{*}}{h_t} - f_t\right) \equiv g_t(h_t) - 1. \tag{24a}$$

It is proportional to the gap between weighted fixed costs abroad and at home. Combining (23a) and (24a) yields the familiar terms-of-trade equation:

$$\xi_t(h_t) = \xi_{t-1} g_t(h_t) \tag{25a}$$

The right-hand side of (25a) is decreasing in h_t and drawn with dotted line in figures 6. There is one intersection point of this curve with $\xi_t(h_t)$ (for $2 < \sigma < 3$ the slope of U-shaped $\xi_t(h_t)$ is below the slope of $\xi_{t-1}g_t(h_t)$). Consequently, equation (25a) has a unique solution $h_t(\xi_{t-1})$ satisfying condition $s_t \in (0,1)$ since it is located between $\overline{h}_t$ and $\underline{h}_t = (2-\sigma) f_t{*}/\psi_t$ turning s_t into zero (for $1 < \sigma \le 2$). The dynamic link $h_t(\xi_{t-1})$ is positive, $h_t'(\xi_{t-1}) > 0$.[13]

As above, the dynamic equation for relative productivity is $\xi_t = \xi_t(h_t(\xi_{t-1}))$. Given ξ_0, and a sequence of the model parameters, this equation determines a dynamic equilibrium path of one-side growth. Existence of asymmetric trading equilibrium with one-side investment implies existence of the whole trading equilibrium path for the two-country world with arbitrary large initial productivity gap. For constant parameters the stage of one-side catching up is finite, because h_t is growing in time and at some period meets the left constraint in (26), which is fulfilled subsequently.[14] There are two stages of growth in this case: in the first one the backward economy is unilaterally reducing the gap, and in the second one both countries are engaged in productivity races. Noteworthy, the pattern of one-side growth represents an extreme case of mean reversion in relative productivity. The advanced country is forced to stagnate temporally to ensure existence of the whole trading equilibrium path.

7. The division of gains from trade

The issue of productivity convergence is related to the issue of division of benefits and losses from economic integration between participating countries. As we have seen, a large initial productivity gap implies slow productivity growth of the advanced country. This can be viewed as a burden of integration borne by the leading economy and offset by an increase of its current market share and national income. Correspondingly, the initially low market share and current income of the backward economy can be viewed as a cost paid for initial acceleration of growth.

---

[13] Equation (25a) implies that $\xi_{t-1} = \xi_t(h_t)/g_t(h_t)$ is increasing in h_t for $1 < \sigma \le 2$ because $\xi_t'(h_t) \ge 0$ and $g_t'(h_t) < 0$. For $2 < \sigma < 3$ this equation is represented as $\xi_{t-1} = \varphi_t h_t^{1-\mu} \frac{\psi_t h_t - (2-\sigma) f_t *}{(f_t * - \psi_t h_t)^2}$ with the right-hand side increasing in h_t.

[14] Growth of the backward economy is positive if and only if the terms-of-trade falls short of the relative fixed costs: $h(\xi_{t-1}) < f*/f$. This is the case for a very large initial productivity gap (ξ_0 close to 0) only if $f* > f\underline{h}$ or $f > \mu\varphi$. This condition is necessary for positive growth of the backward economy. Another condition is necessary for switching to productivity races at some period of time and ensuring that $f*/f$ exceeds the lower bound in (26): $f*/f > ((2-\sigma)f* + \varphi*)/\psi$. For symmetric model parameters this is equivalent to $f > 2\mu\varphi$.

Consider more closely the division of utility gains from trade between countries in the presence of initial productivity gap. The opening of international markets leads to shifting of global output and growth-conducive conditions across the countries. The terms-of-trade improvement raises the advanced country's share in the global market, but reduces opportunities of firms to invest in productivity growth. On the contrary, the lagged country losses in terms of the market share but benefits in terms of the expanding opportunities for growth. The higher is the initial productivity gap, the larger is the scope for such a cross-country shifting of current incomes and abilities to grow.

This is the issue of household utility comparison for different initial values of relative productivity. The log utility in period t is equal to the log difference of expenditures E or E^* and the price index P_t. This specification allows a separation between relative and absolute utility gains from trade and growth. The relative utility gains are determined by the proportion of global demand divided between the countries according to their permanent market shares in global output. The absolute utility gains are mutual and reflected by the common price index that takes a pretty simple form in trading equilibrium:[15] $P_t = p_t r_t (h_t)^{\mu} = p_t * r_t * (h_t)^{\mu}$. This yields domestic household utility:

$$U = \sum_{t=1}^{\infty} \beta^{t-1} \ln(E / P_t) = (1-\beta)^{-1} \ln E - \sum_{t=1}^{\infty} \beta^{t-1} (\ln w_t - \ln x_t + \mu \ln r_t(h_t)) - (1-\beta)^{-1} \ln \theta \, .$$

Similarly, foreign household utility is

$$U* = (1-\beta)^{-1} \ln E * - \sum_{t=1}^{\infty} \beta^{t-1} (\ln w_t * - \ln x_t * + \mu \ln r_t * (h_t)) - (1-\beta)^{-1} \ln \theta \, .$$

To simplify drastically the matter, let us focus on the benchmark case $\sigma = 2$ which is especially suitable for utility comparisons. As we have seen, transition dynamics occurs in this case only during the first period of time, and both economies are on the steady-state path

[15] From (4), (14), $P_t = p_t * (n_t *)^{-\mu} \left((n_t / n_t *)(p_t / p_t *)^{-1/\mu} + 1 \right)^{-\mu} = p_t * (n_t *)^{-\mu} \left((n_t / n_t * h_t) + 1 \right)^{-\mu}$

subsequently. Computing utilities is extremely easy for an equilibrium trajectory consisting of one-period transition stage and the steady-state tail. For symmetric and constant parameters the trading equilibrium of the transition period is characterized as follows:

$$r(h_1) = 2(\psi^* + \psi h_1)/h_1 = 2\psi(1+1/h_1)\,, r^*(h_1) = 2\psi(1+h_1)\,,\ s(h_1) = h_1/(1+h_1)\,,$$

$\Delta x_1 / x_0 = \left(\psi(1+1/h_1) - f\right)/\varphi$. $\Delta x_1^* / x_0^* = \left(\psi(1+h_1) - f\right)/\varphi$. The terms of trade in period 1 are determined by the initial technological gap, $h_1 = h(\xi_0)$, which is supposed to be not too large to meet the condition of positive growth for the advanced economy: $h(\xi_0) > \varphi/\psi$. At subsequent time periods $t \geq 2$ the economies are identical: $\xi_t = h_t = 1$, $s_t = 1/2$, $r_t = r_t^* = \psi$, $\Delta x_t / x_{t-1} = \Delta x_t^* / x_{t-1}^* = f/\varphi - 2$. The condition of positive steady-state growth is also supposed to hold: $f > 2\varphi$.

A widening of initial technological gap shifts the terms of trade in transition period that influence household utilities through four channels. A decrease of $h_1 = h(\xi_0)$ reduces domestic utility through: a) a contraction of domestic market share shrinking domestic expenditure E and b) an expansion of the domestic firm's revenue raising the aggregate price level P_t. The terms-of-trade deterioration has positive influence on domestic utility through: c) a decline of domestic wage and d) a rise of productivity growth, both bringing down the aggregate price level. For $\sigma = 2$ effects b) and c) cancel each other since $\ln w_1 + \mu \ln r(h_1) = \ln s(h_1) + \ln r(h_1) - \ln\theta = \ln 2\psi - \ln\theta$. The residuary influence of the transitory terms of trade on utility is provided by shifts of the market share and productivity growth in period 1. Domestic expenditure is $E = (1-\beta)s_1 + \beta/2 = \left(s_1 + a\right)(1-\beta)$, where $a = \beta/2(1-\beta)$. Productivity in period 1 is $x_1 = x_0\left(1 + (\psi(1+1/h_1) - f)/\varphi\right) =$ $x_0\left(1/s_1 - 1\right)(\psi/\varphi)$, and in subsequent periods it is $x_t = x_1\left(f/\varphi - 1)\right)^{t-1}$. Domestic utility is,

---

$= p_t^*(n_t^*)^{-\mu}(1-s_t)^{\mu} = p_t^* r_t^*(h_t)^{\mu}$. Similarly, $P_t = p_t r_t (h_t)^{\mu}$.

then, equal to $U = (1-\beta)^{-1}\ln(s_1 + a) + (1-\beta)^{-1}\ln(1/s_1 - 1) + const$. The first term indicates the effect of market share in period 1 on domestic expenditure, and the second term relates to the effect of productivity growth in period 1 on the common price index for all periods.

Thus, household utility is reduced to the simple function of period 1 market share indicating interaction of these effects:

$$U = (1-\beta)^{-1}\ln\left(a/s_1 + 1 - s_1 - a\right) + const$$

This function is decreasing in $s_1 = h_1/(1+h_1)$ and, since $h_1 = h(\xi_0)$, in the initial relative productivity ξ_0. By the similar way is shown that the foreign utility is equal to $U^* = (1-\beta)^{-1}\ln\left(a/(1-s_1) + s_1 - a\right) + const$, and increasing in ξ_0.

Consequently, the wider is the initial technology gap, as measured by inverse of ξ_0, the larger is the gain of the backward economy and the smaller is the gain of the leading economy from bilateral trade. The less advanced country benefits from higher growth rate but wastes a certain share in the global market. The growth effect dominates the market share effect, because the former is permanent, while the latter is temporal. An increase of the growth rate in period 1 raises the level of productivity for all succeeding periods. Conversely, a reduction of market share in the transition period does not influence market shares in subsequent periods. By the same reasoning, the leading economy obtains temporal utility gains but incurs permanent utility losses from increasing productivity gap. Such a division of utility gains is stipulated, above all, by the positive dynamic link between the relative productivity and the terms of trade.

8. Concluding remarks

This paper presented a model of trade-driven productivity growth which is induced by a revenue-expanding motive for investment in new technology. The basic feature of the model is the interaction of two terms-of-trade effects. The first one, usually emphasized in trade policy debates, operates directly through changes in relative prices and enforces firms to invest in new technology. The second one operates indirectly, through changes in the numbers of firms, and dominates the first one implying that domestic output is increasing in response to a terms-of-trade improvement. The indirect effect gives rise to the positive dynamic link between relative productivity and terms of trade and generates the empirically relevant patterns of long run productivity growth. The relative productivity and the terms of trade converge monotonously to the steady state, consistent with cross-country evidence. The model, thus, clarifies the causal links discussed in the introduction and allows for interpretation of conditional convergence property in the spirit of modern theory of global trade and growth (e.g. Ventura 1997, Acemoglu, Ventura 2002). International trade is shown to contribute to the widely observed phenomena of rapid growth in the developing countries and slowing growth and even stagnation in the advanced countries.

Monotonous convergence has been demonstrated in spite of the absence of decreasing returns to investment in technology improvement and in production of goods. The model technology lacks capital, for the sake of simplicity and in accordance with the basic goals of the paper. But the model can be extended to include this factor of production and allow trade in the global capital market. In such an extension, the world interest rate supplements the terms of trade as a factor of relative productivity dynamics. The static and dynamic links revealed above are modified appropriately to include the interest rate. Another possible extension is using non-log household utility implying time-varying household expenditures. These generalizations will most likely not alter the critical features of the model underlying the inferences. This matter is left for further research.

As was mentioned in footnote 12, a promising modification of the Krugman-style models is to combine them with quality differentiation. It is possible to modify our model to consider quality upgrading by firms (with national differentiation of quality). The preliminary results show that the key properties of the basic model with productivity growth do essentially preserve, and, moreover, the equilibrium conditions look somewhat more appealing. For example, the latter do not require ruling out the case of high elasticity of substitution (above 3), and therefore make the model applicable to specific industry considerations. Such a modification is also the subject for future research.

The paper does not provide any policy recommendations. The conclusion from utility comparison could potentially add an argument for the less advanced countries to liberalize trade in goods with the advanced ones. But it could also provide an argument for the latter to impose trade barriers protecting domestic producers against too strong terms-of-trade improvement. We would not go as far, and only pay attention to a certain bias in division of gains from trade in favor of less advanced economies. Nevertheless, some extensions of the model allow for policy applications concerning the effects of fixed production costs, entry costs and technology costs on patterns of trade and growth.

Appendix

Proof of Lemma 1. The Lagrangian for the domestic firm's problem can be written as

$$L = ...\beta^t(\pi_t - \lambda_t z_t) + \beta^{t+1}(\pi_{t+1} - \lambda_{t+1} z_{t+1}) + ...$$

where λ_t is the Lagrange multiplier related to the non-negativity constraint (8) in the firm's problem. Differentiating this with respect to x_t implies the first-order condition:

$$\frac{\partial \pi_t}{\partial x_t} - \lambda_t \frac{\partial z_t}{\partial x_t} + \beta\left(\frac{\partial \pi_{t+1}}{\partial x_t} - \lambda_{t+1}\frac{\partial z_{t+1}}{\partial x_t}\right) = 0$$

with $\lambda_t = 0$ for positive investment.

From (7), $\frac{\partial \pi_t}{\partial x_t} = \sigma^{-1}\frac{\partial r_t}{\partial x_t} - \phi_t \frac{\partial z_t}{\partial x_t} = \sigma^{-1}\frac{\partial r_t}{\partial x_t} - \frac{\varphi_t}{\bar{x}_{t-1}}$. Taking into account that the symmetry of local firms is internalized in their decisions, we have $\frac{\partial \pi_{t+1}}{\partial x_t} = -\phi_{t+1}\frac{\partial z_{t+1}}{\partial x_t} = \frac{\phi_{t+1}}{b_{t+1}}\frac{\partial (x_t/\bar{x}_t)}{\partial x_t} = \varphi_{t+1}\frac{\partial (1)}{\partial x_t} = 0$. The first-order condition is reduced to $\frac{\partial \pi_t}{\partial x_t} = 0$ or $\sigma^{-1}\frac{\partial r_t}{\partial x_t} = \frac{\varphi_t}{\bar{x}_{t-1}}$. From (14), $\frac{\partial r_t}{\partial x_t} = \frac{(\sigma-1)n_t * h_t}{(n_t + n_t * h_t)^2 x_t} = \frac{(\sigma-1)s_t(1-s_t)}{x_t n_t}$. Inserting this derivative into the first-order condition and rearranging terms yields:

$$\frac{s_t(1-s_t)}{\theta n_t} - \phi_t z_t = \varphi_t\,. \tag{A1}$$

Combining this with zero-profit condition (13), stating that $\sigma^{-1}(s_t/n_t) = \phi_t z_t + f_t$, implies $(\sigma-1)(1-s_t)(\phi_t z_t + f_t) = \phi_t z_t + \varphi_t$. Consequently,

$$z_t = \frac{\varphi_t - (\sigma-1)(1-s_t)f_t}{\phi_t[(\sigma-1)(1-s_t)-1]} = \frac{(1-s_t)f_t - \mu\varphi_t}{\phi_t(s_t+\eta)}\,.$$

Replicating the same manipulations for the foreign firm yields z_t* in (15).

Proof of proposition 1. The equilibrium number of domestic firms is found by combining (13) and (15):

$$n_t = \frac{\sigma^{-1}s_t}{\phi_t z_t + f_t} = \frac{\sigma^{-1}s_t(s_t+\eta)}{(1-s_t)f_t - \mu\varphi_t + (s_t+\eta)f_t} = \frac{\sigma^{-1}s_t(s_t+\eta)}{(1+\eta)f_t - \mu\varphi_t} = \frac{s_t(s_t+\eta)}{\theta(f_t-\varphi_t)}. \tag{A2}$$

Similarly, $n_t* = \frac{\sigma^{-1}(1-s_t)}{\phi_t * z_t * + f_t *} = \frac{\sigma^{-1}(1-s_t)(\mu-s_t)}{s_t f_t * - \mu\varphi_t * + f_t *(\mu-s_t)} = \frac{(1-s_t)(\mu-s_t)}{\theta(f_t * - \varphi_t *)}$.

This implies a cubic equation on s_t:

$$s_t = \frac{n_t}{n_t + n_t * h_t} = \frac{s_t(\eta + s_t)\psi_t *}{s_t(\eta + s_t)\psi_t * + h_t(1 - s_t)(\mu - s_t)\psi_t}.$$

It has three roots: 0, 1 and the interior one:

$$s_t = \frac{\mu\psi_t h_t - \eta\psi_t *}{\psi_t * + \psi_t h_t}.$$

Substituting this into (A2) yields

$$n_t(h_t) = \frac{s_t(\mu\psi_t h_t - \eta\psi_t * + \eta\psi_t * + \eta\psi_t h_t)}{\theta\psi_t(\psi_t * + \psi_t h_t)} = \frac{s_t\psi_t h_t(3-\sigma)/(\sigma-1)}{\theta\psi_t(\psi_t * + \psi_t h_t)} = \frac{\omega s_t h_t}{(\psi_t * + \psi_t h_t)}$$

and $r_t(h_t) = s_t / n_t = (\psi_t * + \psi_t h_t)/\omega h_t$. The number and revenue of foreign firms is found in the same way.

Proof of proposition 2.

$$\xi_t'(h_t) = \frac{(1-(2-\sigma)^2)\psi_t\psi_t * h_t^{-\mu}}{(\psi_t * - (2-\sigma)\psi_t h_t)^2} - \mu h_t^{-\mu-1}\frac{\psi_t h_t - (2-\sigma)\psi_t *}{\psi_t * - (2-\sigma)\psi_t h_t} =$$
$$\frac{h_t^{-\mu-1}}{\psi_t * - (2-\sigma)\psi_t h_t}\left(\frac{(1-(2-\sigma)^2)\psi_t\psi_t * h_t}{\psi_t * - (2-\sigma)\psi_t h_t} - \mu(\psi_t h_t - (2-\sigma)\psi_t *)\right) > 0$$

if and only if $(1-(2-\sigma)^2)\psi_t\psi_t * h_t > \mu(\psi_t h_t - (2-\sigma)\psi_t *)(\psi_t * - (2-\sigma)\psi_t h_t)$ or

$(1-(2-\sigma)^2)\psi_t\psi_t * h_t > \mu(1+(2-\sigma)^2)\psi_t\psi_t * h_t - \mu(2-\sigma)\psi_t *^2 - \mu(2-\sigma)\psi_t^2 h_t^2$.

Since $1-\mu = -\eta$, $1+\mu = \theta$, $\mu(2-\sigma) = \eta$, this is written as

$$-(\eta + (2-\sigma)^2\theta)\psi_t\psi_t * h_t > -\eta(\psi_t *^2 + \psi_t^2 h_t^2),$$

or, equivalently,

$$(\eta - (2-\sigma)^2\theta)\psi_t\psi_t * h_t > -\eta(\psi_t *^2 - 2\psi_t\psi_t * h_t + \psi_t^2 h_t^2) = -\eta(\psi_t * - \psi_t h_t)^2. \quad \text{(A3)}$$

The right-hand side of this inequality is non-positive for $\eta > 0$ (or $1 < \sigma < 2$). The left-hand side is positive because $\eta - (2-\sigma)^2\theta = \eta(1-(2-\sigma)\sigma) = \eta(\sigma-1)^2 > 0$. Hence, $\xi_t'(h_t) > 0$

for $\sigma \in (1,2)$. The right-hand side of (A3) is non-negative for $\eta < 0$ (or $2 < \sigma < 3$), and the left-hand side is negative. Hence, $\xi_t'(h_t) < 0$ for $\sigma \in (2,3)$.

Proof of proposition 3. Consider equation (25). The lagged relative productivity $\xi_{t-1} = \xi_t(h_t)/g_t(h_t)$ is increasing in h_t for $1 < \sigma \le 2$ because $\xi_t'(h_t) \ge 0$ and $g_t'(h_t) < 0$.

Consider the case $2 < \sigma < 3$. Equation (25) can be represented as $\left(\hat{\xi}_t(h_t)h_t^{-\mu}\right)^2 = \xi_{t-1}h_t^{-\theta}/\vartheta_t$ or $\hat{\xi}_t(h_t)^2 = \xi_{t-1}h_t^{\eta}/\vartheta_t$ where $\hat{\xi}_t(h_t) = \dfrac{\psi_t h_t + (\sigma-2)\psi_t *}{\psi_t * + (\sigma-2)\psi_t h_t}$ is monotonously increasing: $\hat{\xi}_t'(h_t) = \dfrac{\psi_t \psi_t * (3-\sigma)(\sigma-1)}{(\psi_t * + (\sigma-2)\psi_t h_t)^2} > 0$. The lagged relative productivity $\xi_{t-1} = \hat{\xi}_t(h_t)^2 h_t^{-\eta}\vartheta_t$ is monotonously increasing in h_t since $\eta < 0$ for $2 < \sigma < 3$.

Proof of proposition 1a. The number of foreign firms n_t* is found directly from (13): $n_t* = (1-s_t)/\sigma f_t *$. The number of domestic firms is found by combining (13) and (15):

$$n_t = \frac{\sigma^{-1}s_t}{\phi_t z_t + f_t} = \frac{\sigma^{-1}s_t(\eta + s_t)}{(1-s_t)f_t - \mu\varphi_t + (\eta + s_t)f_t} = \frac{\sigma^{-1}s_t(\eta+s_t)}{(1+\eta)f_t - \mu\varphi_t} = \frac{s_t(\eta+s_t)}{\theta\psi_t}. \quad \text{(A4)}$$

This implies a cubic equation on domestic market share

$$s_t = \frac{s_t(\eta+s_t)(\theta\psi_t)^{-1}}{s_t(\eta+s_t)(\theta\psi_t)^{-1} + (1-s_t)(\sigma f_t *)^{-1}h_t}.$$

The interior root is $s_t = -\eta + \mu\psi_t h_t / f_t *$. Inserting this into (A4) yields $n_t(h_t)$ in (17a).

Proof of proposition 2a.

$$\xi_t'(h_t) = h_t^{-\mu}\frac{\left(\psi_t(f_t * - \psi_t h_t) + \psi_t(\psi_t h_t - (2-\sigma)f_t *)\right)}{(f_t * - \psi_t h_t)^2} - \mu h_t^{-\mu-1}\frac{\psi_t h_t - (2-\sigma)f_t *}{f_t * - \psi_t h_t} > 0 \text{ if}$$

$(\sigma-1)\psi_t f_t * h_t > \mu(\psi_t h_t - (2-\sigma) f_t *)(f_t * - \psi_t h_t)$ or $\psi_t h_t / f_t * > s_t(1-s_t)$. This can be written as $s_t + \eta > \mu s_t(1-s_t)$. Rearranging terms yields: $s_t(1-\mu+\mu s_t) + \eta > 0$ or $\mu s_t^2 + \eta(1-s_t) > 0$. This is equivalent to $\frac{s_t^2}{1-s_t} > \sigma - 2$ which is fulfilled for $\sigma \in (1,2]$.

Figure 1. Revenue as a function of productivity

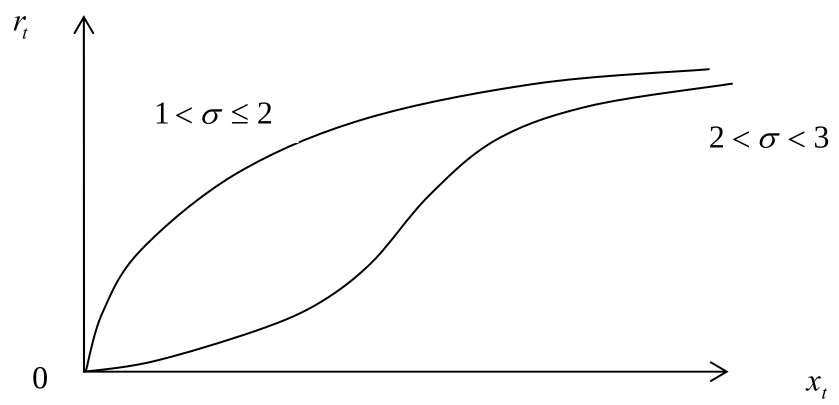


Figure 2. The static link between ξ_t and h_t.

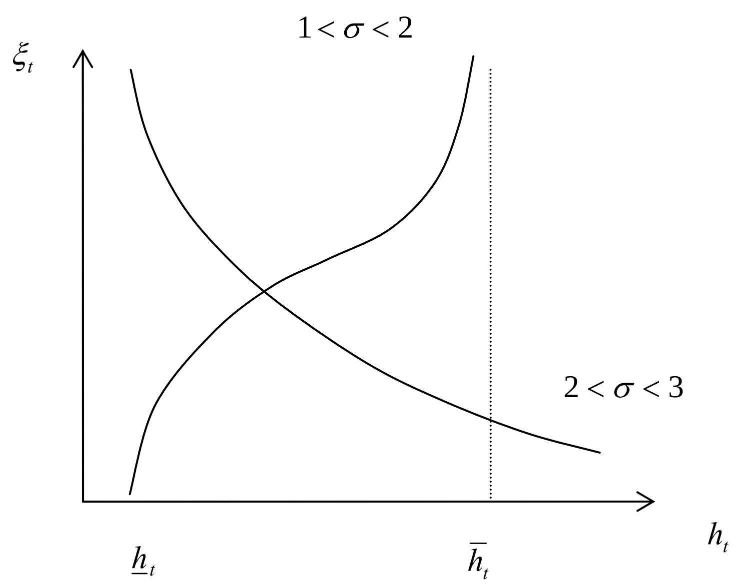


Figure 3. The dynamic link between h_t and ξ_{t-1}.

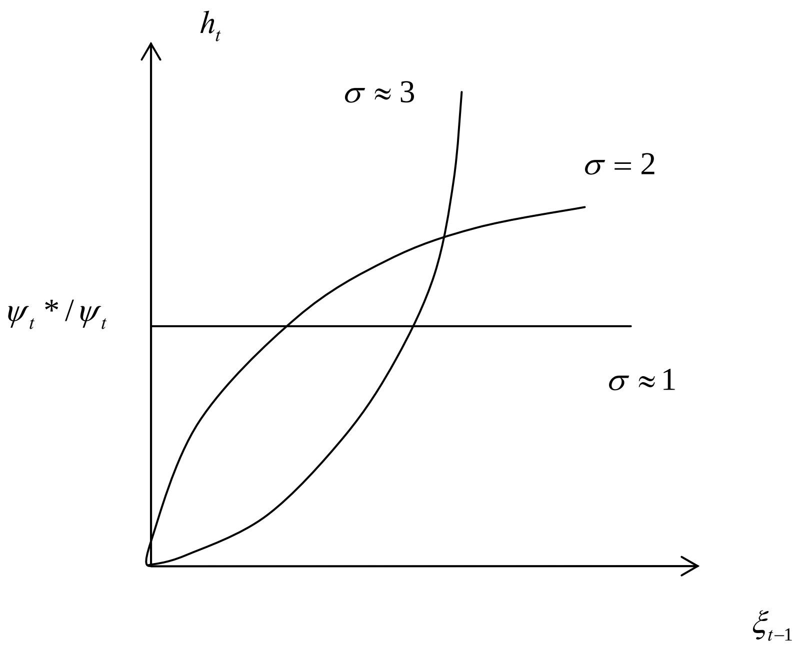
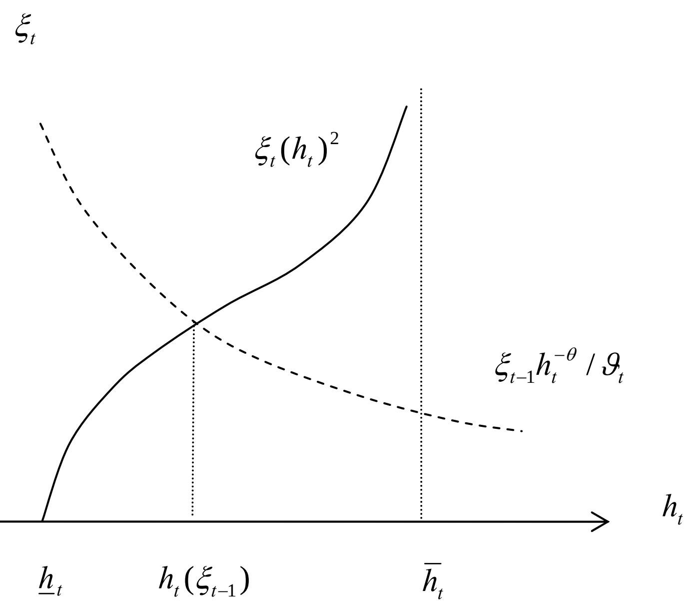
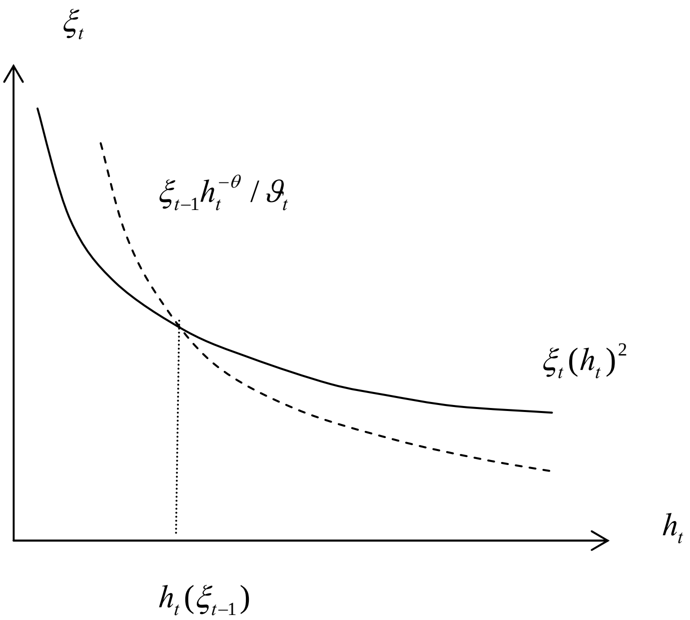


Figure 4. Trading equilibrium

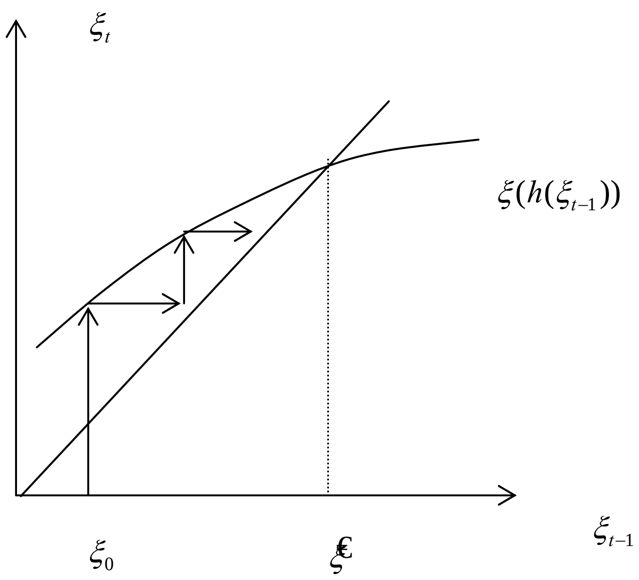


$a) \, 1 < \sigma < 2$

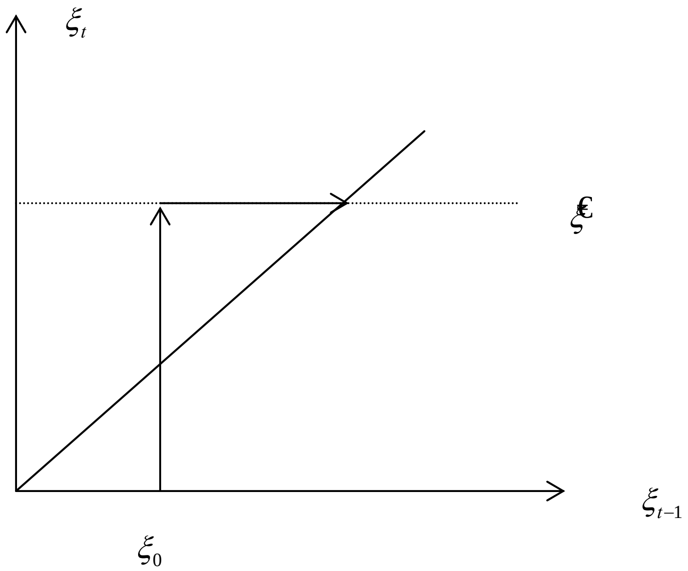


$b)\ 2 < \sigma < 3$

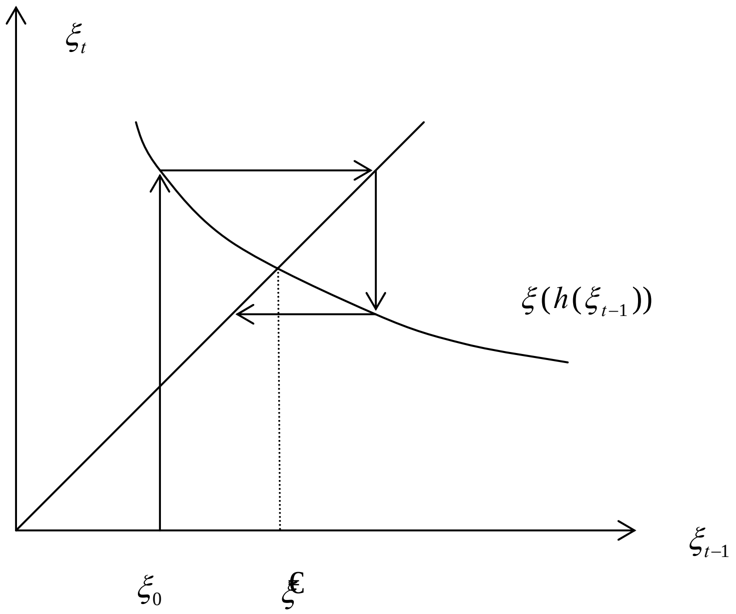
Figure 5. Convergence of relative productivity



$a)\ 1 < \sigma < 2$

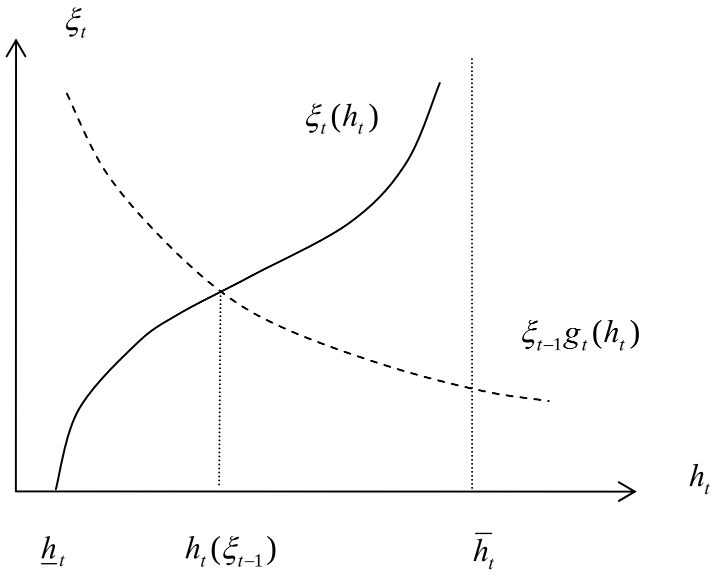


$b)\ \sigma = 2$

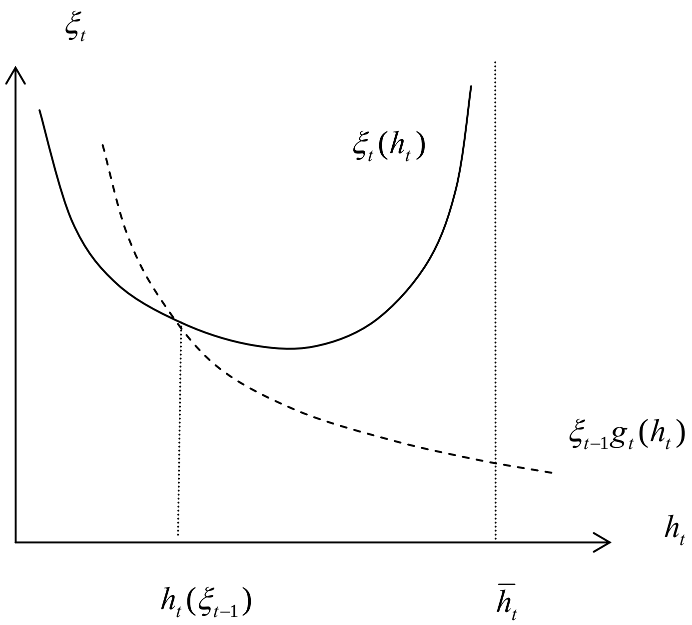


$c)\ 2 < \sigma < 3$

Figure 6. Asymmetric trading equilibrium



$a)\, 1 < \sigma \leq 2$



$b)\, 2 < \sigma < 3$

References

Acemoglu D., Ventura J. (2002), "The World Income Distribution", *Quarterly Journal of Economics*, Vol. CXVII, pp. 659-694.

Aghion P., Howitt P. (1998), *Endogenous Growth Theory*, The MIT Press.

Alquist R., Chinn M. (2002), "The Euro and the Productivity Puzzle: an Alternative interpretation", National Bureau of Economic Research working paper no. 8824.

Armington P. (1969), "A Theory of Demand for Products Distinguished by Place of Production", *International Monetary Fund Stuff Papers*, vol. 1969, pp. 159-178.

Artige L., Camacho C., Croix D. (2003), "Wealth Breeds Decline: Reversals of Leadership and Consumption Habits", manuscript.

Backus D., Kehoe P., Kydland F. (1995), "International Business Cycles: Theory and Evidence", in, Thomas F. Cooley, ed., *Frontiers of Business Cycle Research,* Princeton, Princeton University Press.

Balassa B. (1964), "The Purchasing Power Doctrine: a Reappraisal", *Journal of Political Economy*, Vol. 72, pp. 584-596.

Barro R. (1991), "Economic Growth in a Cross-Section of Countries", *Quarterly Journal of Economics*, Vol. 106, pp. 407-443.

Barro R. (1999) "Determinants of Economic Growth", The MIT Press, Cambridge.

Brezis E., Krugman P., Tsiddon D., (1993), "Leapfrogging in International Competition", *American Economic Review*, Vol. LXXXIII, pp. 1211-1219.

Corsetti G. (2004), "Productivity and the Euro-dollar Exchange Rate", Mimeo, Dallas, FED.

Corsetti G., Martin P., Pesenti P. (2005), "Productivity Spillovers, Terms of Trade and the Home Market Effect", Unpublished manuscript.

Debaere P., Lee H. (2004), ‘The Real-Side Determinants of Countries Terms of Trade: a Panel Data Analysis”, University of Texas at Austin, Mimeo.

Feenstra R. (1994), “New Product Varieties and the Measurement of International Prices”, *American Economic Review,* Vol. LXXXIV, pp. 157-177.

Flam H., Helpman E. (1987), “Vertical Product Differentiation and North-South Trade”, *American Economic Review*, Vol. 84, pp. 810-822.

Grossman G., Helpman E. (1990), “Comparative Advantage and Long-Run Growth”, *American Economic Review,* Vol. 80, pp. 796-815.

Grossman G., Helpman E. (1991), “Innovation and Growth in the Global Economy”, The MIT Press.

Grossman G., Helpman E. (1995), “Technology and Trade”, in: *Handbook of International Economics, Vol. III*, pp. 1279-1337.

Hall R., Jones C. (1999), “Why Do Some Countries Produce So Much More Output Per Worker Than Others?”, *Quarterly Journal of Economics*, Vol. 114, pp. 83-116.

Hummels, Klenow (2005), “The Variety and Quality of a Nation’s Exports”, *American Economic Review*, Vol. 95, pp. 704-723.

Kindleberger C. (1996), *World Economic Primacy 1500-1990*. New York: Oxford University Press.

Krugman P. (1979), “Increasing Returns, Monopolistic Competition, and International Trade”, *Journal of International Economics*, Vol. 9, pp. 469-479

Krugman P. (1980), “Scale Economies, Product Differentiation, and the Pattern of Trade”, *American Economic Review*, Vol. 70, pp. 950-959.

Krugman P. (1981), “Intraindustry Specialization and the Gains from Trade”, Journal of Political Economy, Vol. 89, pp. 959-973.

Lai H., Treffler D. (1999) “The Gains from Trade: Standard Errors with the CES Monopolistic Competition Model”, University of Toronto, mimeo.

Obstfeld M., Rogoff K. (2000), “The Six Major Puzzles in International Macroeconomics: Is there a Common Cause?”, NBER Working Paper No 7777.

Samuelson P. (1964), “Theoretical Notes on Trade Problems”, *Review of Economics and Statistics*, Vol. 46, pp. 145-154.

Ventura J. (1997), “Growth and Interdependence”, *Quarterly Journal of Economics*, Vol. CXII, pp. 57-84.